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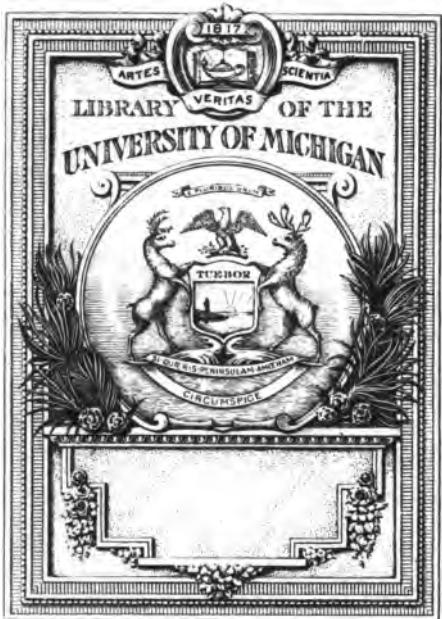
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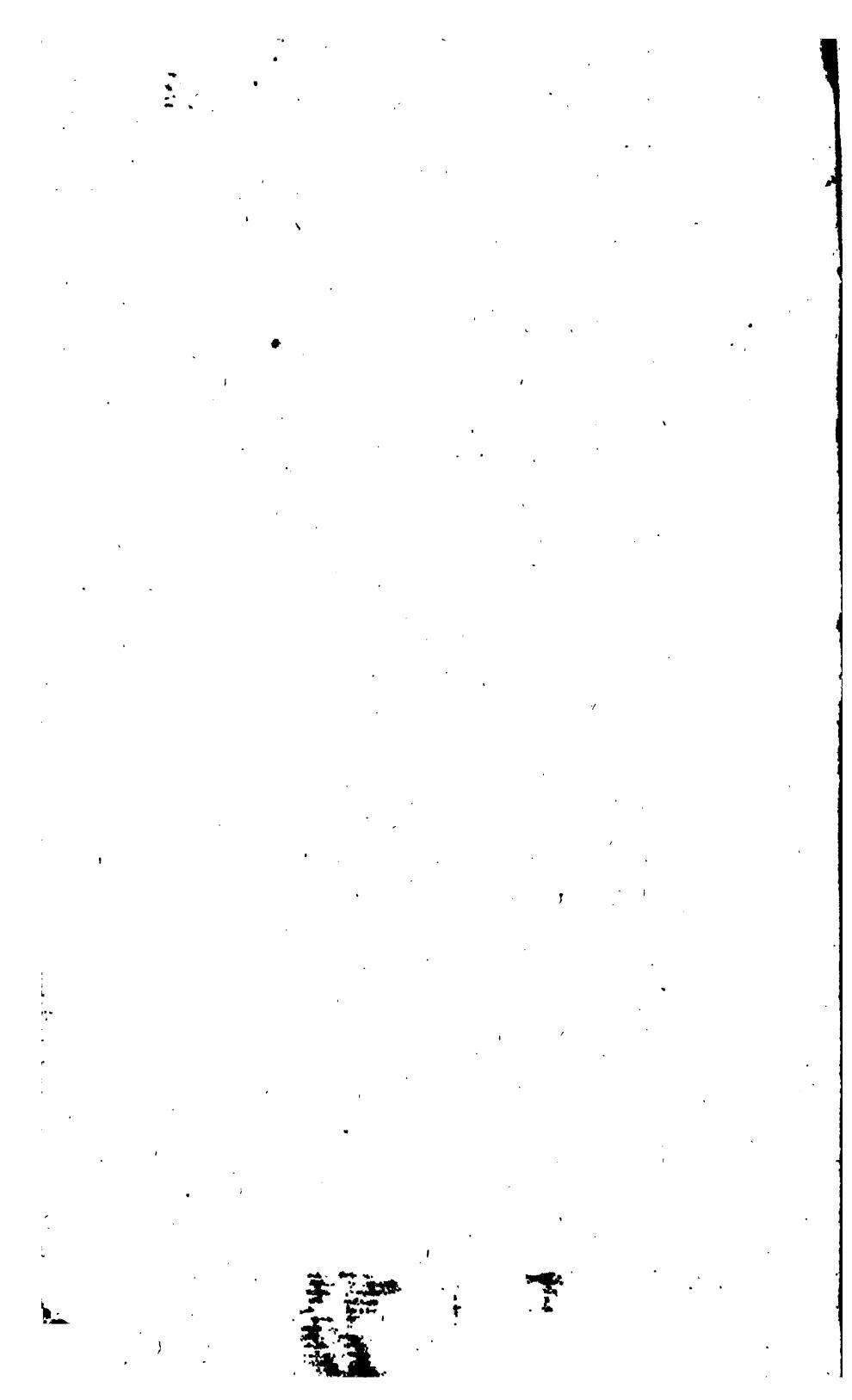
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J. M. W.







William Emerson

A S H O R T

C O M M E N T

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SIR I. N E W T O N's

P R I N C I P I A.

C O N T A I N I N G

N O T E S upon some D I F F I C U L T P L A C E S

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E X C E L L E N T B O O K.

Magnum iter ascendo, —— PROPERT.

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T H E P R E F A C E.

THE PRINCIPIA being a book which is universally read by all the world, that pretend to any degree of philosophical learning; it cannot be improper to explain such passages therein as seem obscure and difficult. For altho' it is written in as clear a stile as can be done in so few words; yet, by reason of its conciseness, and the difficulty of the subjects treated on, many things occur which require some farther explication, especially to young beginners.

Accordingly several mathematical writers have endeavoured to explain some parts or other, of this book, to make them intelligible to common readers, who, without such helps, would find it very difficult to get forward.

The noble subjects this book treats of, being no less than the grand fabric of the world, and the whole system of nature, although comprised in so little a compass, makes it highly deserving of every illustration that can be given it.

The author has clearly shewn in this book, that all the bodies in the world are actuuated by the universal principle of Gravity; which is this, that every body is attracted or impelled towards any other body, by a force which is reciprocally as the square of the distance of the two bodies. On this universal principle he shews, that the motions of all the great bodies in the world are founded.

Several men had written systems of Philosophy before Sir Isaac; but from their ignorance of nature, none of them could stand the test. But his Principles being

P R E F A C E.

built upon the unerring foundation of Observations and Experiments, must necessarily stand good, till the dissolution of nature itself.

This little Treatise was written many years since; for when I studied the Principia, I was frequently at a stop, which obliged me to make calculations here and there, as I went on; and when I had done, I set them down as notes upon these places. Wherein I only meddled with these places, that appeared difficult to me. These notes collected together are the subject of the following Comment. And I have revised the whole, and added several things that seemed wanting. Yet I believe there are some things still behind, which are not sufficiently explained by any Commentator; and especially such as are there laid down without their demonstrations.

As no body is reckoned a Philosopher that does not read the Principia; therefore I thought proper to publish this small Trait, supposing that it may be useful to others, that have a desire to read that celebrated work. What I have further to mention, is this. The passages referred to, and which are to be explained, are not taken from the Latin edition, which would not suit the English reader; but from Motte's translation, and from the first edition thereof, if there happen to be more.

W. Emerson.

A D V E R T I S E M E N T.

Besides the common Algebraic characters which are in use, I make use of this \propto , which signifies a proportion. Thus $A \propto BC$, signifies that A is in a constant ratio to BC , or that A is as BC .

A

A

S H O R T C O M M E N T,

C O N T A I N I N G

**Explanations on Sir ISAAC NEWTON's
Principia, as Translated by AND. MOTTE.**

[**C**OR. 2. to the laws. And therefore if the weight p is to the weight A, &c.] For p will have the same effect as P, if $p : P :: pH \times OL : pN \times$ perpendicular from O on pN . And $P : A ::$ (by what went before) $KO : OL$. Therefore, *ex quo*, if p and A are in equilibrio, it will be $p : A :: pH \times KO : pN \times$ perpendicular from O on pN . Note, the line pN ought to be drawn in fig. 2, and not PL.

[Cor. 4. ib.] This is demonstrated in Keil's Introduction, Theor. 20; as likewise in Lem. 23, p. 127, of this book.

[ib. p. 28, *sub finem*; but the distance between these two centers,] that is, between the center of the two, and the center of all the rest. Further, the actions of all the bodies may be considered as the sum of the actions of every two; and then the case will be plain.

[Sch. to cor. 6, p. 33, then will ST represent] For if $RV =$ retardation of describing $2RA + 2VA$, ST will be the retardation of describing $\frac{1}{4}$ of that, or $\frac{1}{2}RV + \frac{1}{2}VA$, that is SA or FA; and therefore the body falling from S in the air, or from

A 3

T in

A C O M M E N T O N

Fig. T in vacuo; will have, nearly, the same velocity in A; the same of sA or tA ; for ascending to s in the air, or ascending to t (or descending from t to A) in vacuo has the same velocity in A.

[ib. p. 38, but if they are turned aside by the interposition] This is plain by Prop. 16, Mechanics.

[ib. And in like manner, &c.] By the same Prop. as before.

B O O K I.

S E C T. I.

[LEM. 10.] For let AE be divided into an infinite number of equal parts, any part, as $D'x$ by the velocity acquired in the time AD (that is, $DxDB$), is as the space described in that little part of time D' (for the spaces are as the times x by the velocities); and the sum of all these products or areas, that is, the areas ADB, AEC, are as the whole spaces described in the times AD, AE; but these areas are as AD^2 and AE^2 , by Lem. 9.
 [ib. cor. i. — to the bodies, and measured —] that is, and the said errors measured by the distances of the bodies, &c.

[LEM. 11. have a finite curvature] These words exclude those curves whose radius of curvature is infinitely small, or infinitely great.

[ib. Schol.] All this may be universally demonstrated after this manner. Let AB, AF be two paraboloids. Let, latus rectum, of AF $= a$, of AB $= b$. AC $= z$. AE $= x$. CB, EF, or AD $= y$. And let m be any affirmative index, and suppose $a^m x = y^{m+1}$. And $b^{m+n} z = y^{m+n+1}$.

$$\text{Then } a^{\frac{m}{m+1}} x^{\frac{1}{m+1}} = y = b^{\frac{m+n}{m+n+1}} \times z^{\frac{1}{m+n+1}}.$$

And by involution, $a^{m \times m+n+1} x^{m+n+1} = y^{m+n \times m+1} z^{m+1}$. And therefore, $a^{m^2+mn+m} x^m : b^{m^2+mn+m+n} : : z^{m+1} : x^{m+1}$. But, because x is infinitely small, therefore when n is affirmative, the first term is infinitely less than the second; and therefore z^{m+1} is infinitely lesser than x^{m+1} ; but if

Fig. n is negative, the first term is infinitely greater than

1. the second, and therefore x^{m+1} is infinitely greater than x^{m+1} , or x than x ; that is, in case the first DB is infinitely less, and in case the second, infinitely greater than DF. But DB, DF are as the curvatures of AB, AF, therefore, &c.

S E C T. II.

2. [Pr. 1. cor. 4. as the versed sines, of arcs] described in equal times, tending to the center of force, and bisect the chords; let $AB = BC$, and Bd be \perp to AC, then $Ad = dC$. And when the arch AC is diminished to infinity, e coincides with d ; and consequently $Ae = eC$, or BeS bisects the chord AC. Complete the parallelogram ABCf, and $eB = \frac{1}{2}Bf$. Note, he calls that the versed sign of an arch, which is commonly called the versed sine of half that arch.

[Pr. 4. cor. 2.] For the forces $\propto \frac{\text{velo.}^2}{\text{rad.}}$
 $\frac{\text{vel}^2 \times \text{rad}}{\text{rad}^2} \propto \frac{\text{rad}}{p.\text{times}^2}$; all the other corollaries
(except the last) depend on this.

[ib. cor. 8.] Let R = radius of curvature D = distance from the center of force. Then will the p.time $\propto R^n \propto D^n$ (by similar position), and the areas \propto velocit es $\propto R^{n-1} \propto D^{n-1}$. And therefore the force $\propto \frac{1}{D^{n-1}}$: and the contrary.

3. [ibid. cor. 9.] Take the arch Bd infinitely small, and let BF be described by the revolving body, in the same time that a body falls from B to E by an uniform centripetal force, which it has at B; then cd is the space fallen in the time of describing Bd . But Cd or $Bn : BE ::$ (as the squares of the times, that

THE PRINCIPIA

that is, as) $Bd^2 : BF^2 :: \frac{Bd^2}{BA} : \frac{BF^2}{BA}$. But $Bd = \frac{BF^2}{BA}$ Fig. 3.
 $\frac{Bd^2}{BA}$. Therefore $BE = \frac{BF^2}{BA}$. And $BE : BF :$
 $BA ::$.

[ib. Schol. as the square of the length applied to the radius;] for, the number of reflections is \propto velocity or length directly, and the radius reciprocally.

[Pr. 7. cor. 2. in the same periodical time] Let 4.
 ac be \parallel to RP , and da \parallel to PS . Then if the periodic times be equal, the areas generated in a given infinitely small time must be equal, that is, the velocities round R and S must be reciprocally as (the \perp 's on PG from R and S , that is, as) RP and SG . And supposing a, c, d , to coincide in P , the force round R to the force round S is in the complicate ratio (of ac to ad , or) SG to SP , and the squares of the times of describing a given arch, that is reciprocally as the squares of the velocities, that is, as SG^2 to RP^2 . Therefore the force round R to force round S :: is as SG^2 to $SP \times RP^2$, when the periodical times are equal.

Or thus, let p be the place of the body when the tangent pg is \parallel to the line RS . Then the velocities round R and S , in the place p , will be equal; for the small areas are equal, and their heights are equal, by reason of the parallels RS, pg . Draw TV, tv , and then by similar triangles $Sp^2 \times Pv^2 = Sg^2 \times Pt^2$. And $Sp^2 \times Pv^2 = SG^2 \times PT^2$. Then by this prop. force round S , in p : force round S , in p :: $Sp^2 \times Pv^2$ or $\frac{Sg^2 \times Pt^2}{Sp^2} : SP^2 \times PV^2$ or $\underline{\underline{SG^2 \times PT^2}}$.

SP

And force round S , in p : force round R , in p :: $Sp ; Rp$ or Sg .

Also

A C O M M E N T O N

Fig. Also force round R, in p : force round R, in p ::
 4. $RP^2 \times PT^3 : RP^2 \times pt^3$ or $Sg^2 \times pt^3 :: ex equo$, force
 round S, in P : force round R, in P :: $RP^2 \times PT^3 : SP^2 \times PT^3$
 $\therefore SG^2 \times PT^3 :: SP \times RP^2 : SG^2$.

SP

[Cor. 3, in the same periodic time] for then the infinitely small and equal areas will be described in equal times in P, and both these areas and the forces will be the same, as in a circle of the same curvature with the orbit at P, and therefore the forces are the same as in the foregoing Corol.

5. [Pr. 8. Sch.] let ApD be an ellipsis, AP a circle. Then $rn : Rn :: (pm : Pm ::) qn : Qn$. And by division, $qr : QR :: (qn : Qn ::) CD : CA$. The force in the ellipsis $\propto \frac{qt^2 \times Sp^2}{qr}$ reciprocally =
 $(\text{because } qr = \frac{QR \times CD}{CA}) \frac{QT^2 \times CA \times Sp^2}{QR \times CD} =$
 $(\text{because } \frac{QT^2}{QR} = \frac{2Pm^3}{CA^2}) \frac{2Pm^3 \times Sp^2}{CA \times CD} = (\text{because } Pm^3 = \frac{pm^3 \times CA^3}{CD^3}) \frac{2pm^3 \times CA^2 \times Sp^2}{CD^4}$. Therefore (because CA, Sp and CD are given) the force in the ellipsis is $\propto pm^3$ reciprocally. But in the hyperbola and parabola (where CA is negative or infinite) these lines are still given, and therefore the force in any conic section is reciprocally as mp^3 .

[Pr. 9. will be changed—] that is, $\frac{QT^2}{QR}$ is every where the same ratio, viz. $\propto SP$.

[ib. second way.] for PV is (by reason of the given angle at P) as the radius of curvature, that is (by reason of the similarity of the parts of the figure PQ) as SP.

[Pr. 10. second way. Add the rectangle nPv .] for $Qv^2 + nPv = Qv^2 + TP + Tv \times TP - Tv = Qv$

$= Qv^2 + TP^2 - Tv^2 = QT^2 + TP^2 = \text{square of Fig.}$
the chord QP ; and $Pv \times nV + nPv = VPv$. 5.

And if the line QV be drawn, and a circle thro' the points PQV ; the triangles PQv , and PQV will be similar (the $\angle QVP$ being $= QPR = PQv$), and therefore $Pv : PQ : PV$.

[Pr. 10. Sch. in the ratio of the distances from the center] for the fluxion of the ordinates is augmented or diminished in the same ratio, and that is as the force.

S E C T. III.

[Pr. 13, cor. 2.] For, in the demonstrations of Prop. 11, 12, and 13, QT^2 is always equal to $QR \times \text{latus rectum}$.

[Pr. 16, cor.] The four first corollaries are general, and agree to all conic sections; the sixth corollary belongs to one and the same parabola.

[ib. cor. 6, it is more variable] that is, in the ellipsis, the ratio of the velocity at a less and greater distance, is greater than the ratio of the square roots of the greater and lesser distance; in the hyperbola 'tis greater; for the ratio of the greater and lesser perpendiculars (which is the same with this ratio of the velocities) is greater than the ratio of the square roots of the greater and lesser distances in the ellipsis, and lesser than it in the hyperbola. For in the ellipsis the perpendiculars in the greatest and least distances are the same with these distances; and in the hyperbola, the greatest perpendicular possible is that from the focus on the asymptote, and the least, the distance to the vertex. Also (by Conics) the perpendicular $Sy \propto \sqrt{\frac{SP}{HP}}$ (fig. 3.)

See Clark. (also cor. 7.)

[Pr. 17. yet greater velocity] then PH (and PK) will be negative.

[ib.

Fig. [ib. cor. 2.] The velocity in a circle is = velocity acquired by falling thro' $\frac{1}{2}DS$, by the given centripetal force.

S E C T. IV.

[Pr. 18, 19, 20.] two given points or right lines.

[Pr. 20, case 4. But because of the similar triangles VSH, vsb,] For (by similar Δ's SVP, sbq) $SV : SP :: sb : sq ::$ (by sim. Δ's, svb, spq) $sv : sp$. And (by the similar Δ's SHP, sbp) $SP : SH :: sp : sb$; ergo $SV : SH :: sv : sb$; and the angle VSH = psq = vsb.

[Pr. 21. three given points or right lines.] If three tangents be given, you have 3 points Y, from which 3 equal right lines as YH are to be drawn to H, by case 3d of the last Lemma. If 2 tangents and a point P, there will be given two points Y, from which two equal lines are to be drawn to the focus H, and a third point P, from which PH is to be drawn; by case 2d Lem. In the hyperbola, it is $PH - YH = SP$. If three points P be given, it is done by case 1 Lem.

S E C T. V.

[Lem. 17. in given angles] each to each respectively is the same invariable angle, tho' they are not all equal.

[Lem. 18. and so (by supposition)—] for p and b are in the curve, and the rectangles of the lines from p and P are in the given ratio.

[ib. Sch. if p happens to be in a right line] For if $rpq : spt :: RPQ : SPT$. And P be placed in pB; then by similar triangles $pq : PQ :: (pB : PB ::) pt : PT$. And $rp : RP :: (Cp : CP ::) ps : PS$. And by multiplying you have $rpq : RPQ :: spt : SPT$; which agrees with the Lemma. But if P is placed out of pB, as at n; then because the ratio

ratio $\frac{RnQ}{SxT}$ is greater than $\frac{RPQ}{SPT}$ it is also greater Fig. 6.

than $\frac{rpq}{spt}$, which is against the hypothesis.

If p is placed in DC, then rpq will be = 0, and therefore $RPQ = 0$; and P will also be placed in DC.

[ib. If the two opposite angles] The Δ's PCR, PBT are similar, for the angles at S and T are right, and C = B, being on the same arch PD. Therefore $PR : PC :: PT : PB$. And the triangles PBQ, and SCP are similar, for the angles at S, Q are right, and $SCP = PBA$. Therefore $PC : PS :: PB : PQ$; and, ex equo, $PR \times PQ = PS \times PT$: and, e contra, if $PR \times PQ = PS \times PT$; the locus of the point P is a circle. 7.

And if these lines are not perpendicular, yet since their lengths will be reciprocally as the sines of the angles, it will follow that $PQ \times PR : PS \times PT :: \text{fine of } S \times \text{fine of } T : \text{fine of } Q \times \text{fine of } R$, and the contrary.

[ib. and one or two] If the point B be supposed to move towards A and at last to coincide with it, so that AB become a tangent at B, the Lemma will still hold. And if B pass beyond A, then the figure will be converted into this BECDEAB; the Lemma will still hold as before. And if B move to an infinite distance, then DB, AB will be parallels; and C also, then DC, AC will also be parallel; and also the conic section passing thro' A and D, will pass on infinitely towards C and B. 8.

[Lem. 19.] This may be resolved as Prob. 12, in the Universal Arithmetic.

[Lem. 20.] Here is another figure relating to this Lemma (fig. 9.) 9.

[Lem. 21. Therefore (by Lem. 20) the point D] for the angles CPR, BPT, CPB are given, and therefore the lines PT, PR (to which the sides AQ,

Fig. AQ, AS of the parallelogram are parallel) are given by position (see fig. above).

10. Here is another figure of Lem. 21. (fig. 10)

This Lemma is the same with Prob. 53, of the Universal Arithmetic.

11. [Prop. 22.] This is the same with Prob. 55, in the author's Universal Arithmetic.

[Pr. 23. case 1.] This is the same with Prob. 56, of the Universal Arithmetic, or Prob. 57.

[Pr. 23. case 2. HA² will become to Al²] for let n be a point in the conic section infinitely near a, thro' both which the line ib passes; then ib is a tangent at a. Then (Cor. 4. Pr. 44. B. I. my

Conics) it will be, bgd : bbd :: pgc : xby = $\frac{bbd \times pgc}{bgd}$,

and bna or ba² : ian or ia² :: xby or $\frac{bbd \times pgc}{bgd} : pic$

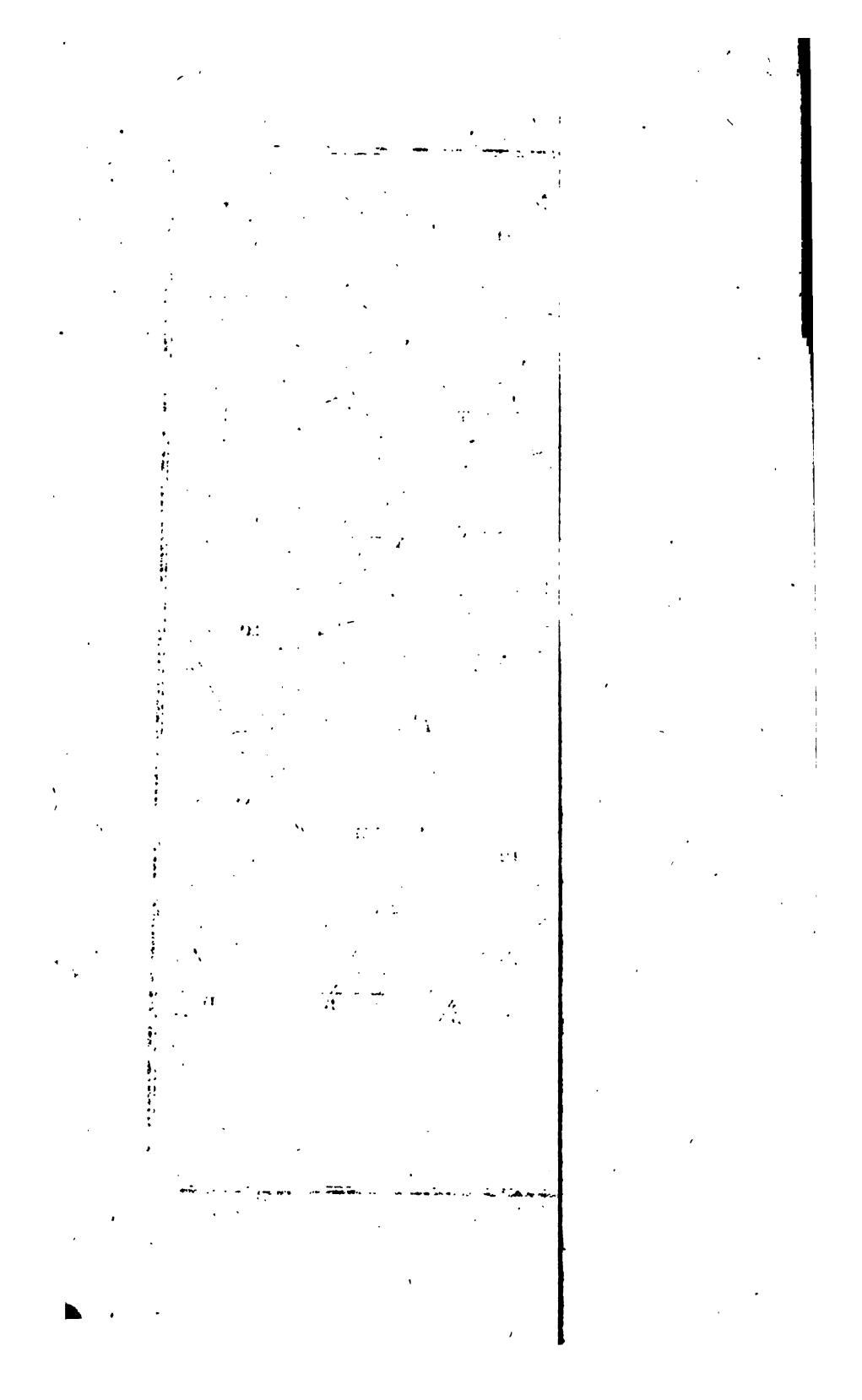
:: bbd × pgc : pic × bga.

12. [Pr. 24. — will (by the properties of the Conic Sections)] because any tangent may be supposed to cut the curve in two points infinitely near each other, therefore (by Cor. 4. Pr. 44. B. I. Ellipsis) these proportions follow. Also draw kw || to ga, to intersect pg, and let nk² = tkw. Then tkw or nk² : ba² :: dkb : bbd :: kr² : br². And nk : ba :: kr : br. And n, r, a, fall in one right line. Again, kn² : ga² :: kp² : gp². And kn : gn :: kp : gp. Whence n, p, a fall in one right line. And therefore the points p, r, a are in one right line.

13. [Lem. 22. Thus any right lines converging] Let the lines be kg, kb. Draw OB, Od, and pe || to NB. Let m, q be the projected points of f and b. Then (because in the point B, OB = OD,) mn (or Bn — Em) = fr. Also qp (or ep — eq) : bg (or Dg — Db) :: Oe : Od :: AB : AD :: fr or mn : kg. Therefore qp = mn, therefore pn, qm (which are the projected lines of gr, bf,) are parallel.

[ib.

750



[ib. we shall have the solution required] For the Fig. figure hgi supposed now to be given, may be transformed into the first figure HGI, by making as Od to dg, so is OD to DG parallel to the radius AO. 13.

[ib. For as often as two conic sections] For these conic sections being transformed into simpler ones, give the point of intersection; and thereby is had an ordinate drawn from that point of intersection in the transformed curves, corresponding to the intersection of the given curves.

[Pr. 25.] Let KG, KH be two tangents meeting in K, yx the third tangent meeting ba in y. Let the lines KH, KG be projected in $ki\ lb$; and yb , yx into kl , ib ; then you have the parallelogram $bikl$; then proceed according to the proposition. 14.

[ib. For by the properties of the conic sections] by cor. 4. Pr. 44. Conics I.

[ib. But according as the points] This is plain, 15. from the nature of the ellipsis and hyperbola; and 16. the figure cannot be a parabola, by reason of two parallel tangents ib and kl .

[Lem. 23.] The Lem. is universal, as will appear by applying the demonstration to this fig. 17.

[Lem. 24, from the nature of the conic sections] by Pr. 46. B. I. of my Conic Sections.

[ib. cor. 1.] This holds as well when the tangent FG is on the other side. 18.

[Lem. 25. also KH is to HL] by cor. 1. Lem. 24, for the tangents FH, LH cut the parallel tangents ML, IK in F and K.

[Lem. 25. cor. 2.] This holds as well when qe 19. is on the other side of the figure; for, in all cases (by cor. 1.) $KQ \times ME$ is given wherever the points Q, E fall, as suppose in q, e : for (by cor. 1. Lem. 24.) $Bq : AM$ or $BK :: eI : eM$. And by division $Kq : BK :: MI : eM$. And $Kq \times eM = BK \times MI =$ (by the prop.) $KQ \times ME$; and $KQ : Me :: (Kq : ME ::) Qq : Ee$.

[ib.

A C O M M E N T O N

Fig. [ib. cor. 3.] for since $eM : ME :: QK : Kq$.
 19. Therefore (by Lemma 23.) if the right lines eQ , MK , and Eq be drawn, the points of bisection will be placed in a right line given in position.

[Pr. 27. Sch. describe the circle BKGC.] for the angle BKC (which is equal to the sum of the given angles PBK and KCP) is always given.

[ib. and when those other legs CK, BKJ For it must be observed, that when the lines BP, CP, touch the curve at an infinite distance, that these lines are parallel to one another, and to the asymptote. Then to know the position of the asymptote; as the lines Bk , Ck revolve round the circle, the intersection k will sometime fall into the line MN, as at the point N, then BN is parallel to one asymptote. For the intersection must necessarily fall in the line MN. And for the same reason the line BM will be parallel to the other asymptote. And therefore if the angle between them be bisected by the line OH (which is done by the perp. OH), then that line is the greater axis, or parallel to it.

[ib. Sch. But the squares of the axes] For the angle NBM = angle between the asymptotes = NLM, and NLH = half the angle of the asymptotes. Therefore LH is to HN, as the transverse to the conjugate. And the squares of these axes, are as LH^2 to HN^2 , or as HN^2 to HK^2 , that is, as LH to HK.

20. [ib. There are also other Lemma's] For if the sections are similar, and in similar position, and concentric; the tangent acb in c , is parallel to the tangent dxe in x , and therefore ab the ordinate is bisected in c the point of contact.

[Lem. 26. cor.] This is the same with Prob. 32, of the Universal Arithmetic.

[Lem. 27. cor. in the construction] for then $IH : HF :: (IX : XY ::) ib : bf$. And $IH : HG :: (iL :$

B. I. THE PRINCIPIA.

17

$iL : LM :: ib : bg$. Or, if it be made as $iL :: Fig.$
 $LM :: IH : HG$, it will be $ib : bf :: IH : HF$; 20.
 therefore on the contrary, if it be $ib : bf :: (iE :$
 $EV :: iX : XY ::) IH : HF$. It will be $iL : LM ::$
 $IH : HG$, which comes to the former construction.
 For the solution of Prop. 22, 23, 24, 25, 26, 27;
 see Prop. 70, 71, 72, 73, 74, 75, B. III. my
 Conic Sections.

[Pr. 29. Sch.] Make also KA to AS, and LT to 21.
 AT, as HG to GF, and draw MS, NA. Then
 the figures SAKM and ATLN are similar to
 FGHI, and since three of the angles S, A, K or
 A, T, L are in the proper lines CB, ED, DB; if
 the fourth angle M or N, was in the fourth line
 EC, the problem would be rightly constructed.
 Therefore it is plain, its place can be no where,
 but where the line MN intersects EC as at i , which
 is the place of the angle I.

Now we are to prove that PQ cuts BA in f
 where F is to be placed. The triangle FGI is simi-
 lar to PE i (by construction), and suppose them
 similar to fgi ; then the triangles Pfi and Egi are
 also similar; for the angles at i are equal, and the
 sides, about these angles proportional; therefore
 the angle $Egi = Pfi$, and since $goi = Qof$, oQf
 or PQE will be $= fig$. So that to have fig similar
 to FIG, PQF must intersect Eg in Q, to make the
 $\angle PQE = FIG$, and the rest follows of course.

S E C T. VI.

[Pr. 30, Cor. 1.] For the times are as the areas,
 that is, as $\frac{4}{3}GH \times AS$ to $\frac{2}{3} \times AS \times 2AS$.

[ib. Cor. 2.] For wherever the point P falls, viz.
 infinitely near A, still $\frac{4}{3}GH \times AS$ (\equiv area APS)
 $\equiv \frac{1}{2}AP \times AS$. And $8GH = 3AP$. Therefore
 GH or the velocity of H : AP or the velocity of P
 $:: 3 : 8$. But the velocity of H is every where

B

the

A C O M M E N T O N

Fig. the same, for always $\frac{1}{2}GH \times AS = \text{area } ASP$;
And $GH \propto ASP \propto \text{time of describing AP}$.

[ib. Cor. 2.] For AP is the chord of a circle passing thro' A, S, P; and whose center is H.

[Pr. 31. as GK the difference] For when F comes to touch the line GH, the point A will be distant towards G, from the line FQO (which will then be \perp to GH) by the sine of AOR or AOQ; and is then at L; therefore $GK = \text{arc } GF - \text{sine of the arc } AQ$. And GK is as the time, and so it was in the construction; therefore the point P is rightly found.

22. [ib Sch. but since] This is demonstrated in Keil's
 23. Astron. lectures, pa. 289, 297; or thus. Let $AON = N$. Since $(AB : SH ::) OQ : OS :: 57.29578 : B$. Therefore $B = OS$ in degrees of the circle AQ. And since $(\text{rad. : sine of } AOQ ::) SO : SF :: B : D$. Then will $SF = D$ in degrees. Let q be the true place of the body; Q the assumed place. Now since the time is as the area $SAQ = OAQ + SOQ$
 $= \overline{QA + SF} \times \frac{OA}{2}$; therefore the time is as $AQ + SF$. And therefore nearly as $AQ + D$, but accurately as $Aq \pm SE$. Take $N\phi = D$. Now $OE : OQ :: (LE \text{ or } SE - SF \text{ or }) Nq \pm N\phi \text{ or } qp \text{ or } Q\phi - qQ : Qq$. And $Q\phi : Qq :: QE : OQ$, by composition, and because $QE = OQ \pm OE$. But by construction $OQ = \frac{OS \times L}{R}$. And rad. (R) : cos. of $AOQ :: SO : OF \text{ or } OE$, therefore $OE = \frac{SO \times \cos. \angle OQ}{R}$. Wherefore $Q\phi : Qq :: (QE =)$
 $\frac{OS \times L}{R} \pm \frac{SO \times \cos. AQ}{R} : \frac{OS \times L}{R} :: L \pm \cos.$
 $AQ : L$. But $N - AOQ + D = (AN - AQ + N\phi =) Q\phi$. Therefore $Qq = E$, and $AOq = AOQ + QOq = AOQ + E$, nearly, and therefore

B. I. THE PRINCIPIA.

19

fore E is rightly found. And repeating the same Fig. work with these new angles, there will be found 22. the angles F, G, H, I. 23.

[ib. the area AIKP will be given] by Prop. 87. cor. 2. and Schol. my Conic Sections, B. II. Also $\Delta AIKP \equiv \Delta OPA$, by cor. 1. Prop. 86. ib.

Also area described $\equiv \Delta ASQ \equiv \Delta ASP + \Delta PSQ$
 $\equiv \Delta ASP + PQ \times \frac{1}{2}SN$ (nearly) $\equiv A$ (by supposition) therefore $A - \Delta ASP \equiv \frac{1}{2}SN \times PQ$;
and $PQ = \frac{2A - 2\Delta ASP}{SN}$, nearly.

[ib. and by such computations. — But the particular calculus] with the radius 1 and center H describe the circle RSE; and draw SP, Sp, HPT, Hpf. Let fall the perpendiculars pt, pu, upon the lines HP, SP; which will be equal, because the angles pPt (HPB), and pPn, are equal.

Let the arch RT $= z$. Sine TQ $= s$, cof. HQ $= x$, SP $= y$, HP $= v$, AO or OB $= a$, SO or OH $= n$, OD $= c$. T = mean anomaly, l = latus rectum.

By similar sectors, HT (t) : HP (v) :: Tf (z) : pt or pn $= v\dot{z}$. And the area $\frac{SP \times pu}{2} = \frac{vy\dot{z}}{2} \equiv$ fluxion of BPS. But (cor. 2. Prop. 23, ellipsis)
 $vy = cc + \frac{nn}{cc} \times PM^2$, and (cor. 3. Prop. 72, ib.)

HP or v $= \frac{cc}{a + nx}$, and rad (1) : PH $\left(\frac{cc}{a + nx} \right)$
 $:: S : PM = \frac{cc s}{a + nx}$, therefore $vy = cc + \frac{nncss}{a + nx}$

and area $SPp = \frac{1}{2}cc\dot{z} + \frac{\frac{1}{2}nncss\dot{z}}{aa + 2anx \&cc.} = \frac{1}{2}cc\dot{z} + \frac{nncss\dot{z}}{2aa} - \frac{n^2c^2s^2x\dot{z}}{a^3}$. But $s\dot{z} = -\dot{x}$, and $x\dot{z} = \dot{s}$,

B 2

and

20 A COMMENT ON

Fig. and $\frac{cc}{a} = \frac{1}{2}l$; therefore the area $SPp = \frac{1}{4}alz -$
24.

$$\frac{\frac{1}{4}nnlzs^2}{a} - \frac{n^3ls^3}{2aa}, \text{ and the fluent } BSP = \frac{alz}{4} - \frac{nnl}{4a}$$

$$\times \text{area EHQT} - \frac{n^3ls^3}{6aa}; \text{ and corrected, } BSP =$$

$$\frac{alz}{4} + \frac{nnl}{4a} \times \text{area RTQ} - \frac{n^3ls^3}{6aa}. \text{ But } RTQ = \frac{1}{2}z$$

$$- \frac{1}{2}xs. \text{ Whence } BSP = \frac{1}{4}alz + \frac{\frac{1}{4}nnl}{a} \times \frac{\frac{1}{2}z - \frac{1}{2}xs}{}$$

$$- \frac{n^3ls^3}{6aa} = \frac{2aa + nn}{8a} lz - \frac{nnls}{aa} \times \frac{ax}{8} + \frac{nss}{6}. \text{ And}$$

$$\text{dividing, } z - \frac{8nns}{2a^3 + ann} \times \frac{ax}{8} + \frac{nss}{6} = T,$$

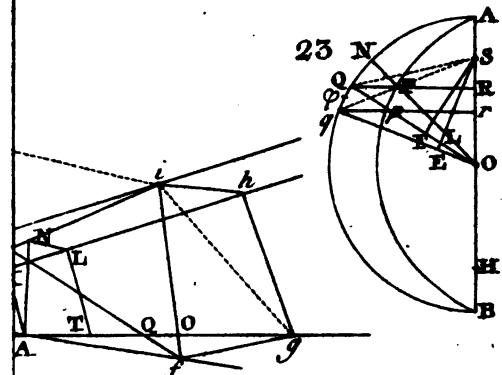
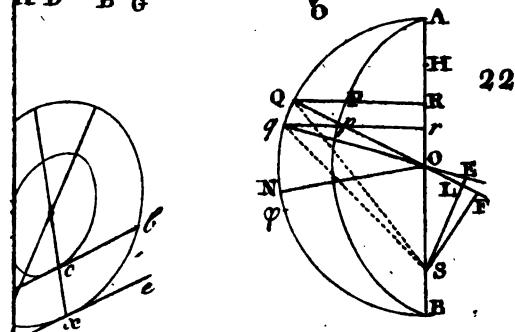
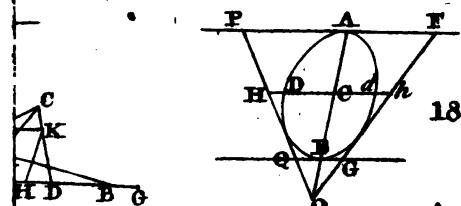
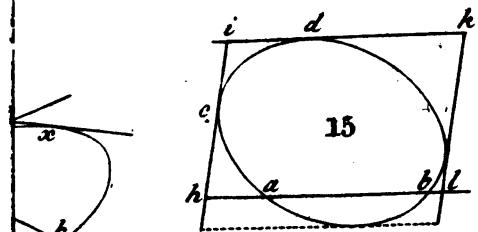
$$\text{the mean anomaly. Whence } z = T + \frac{nn}{4aa + 2nn}$$

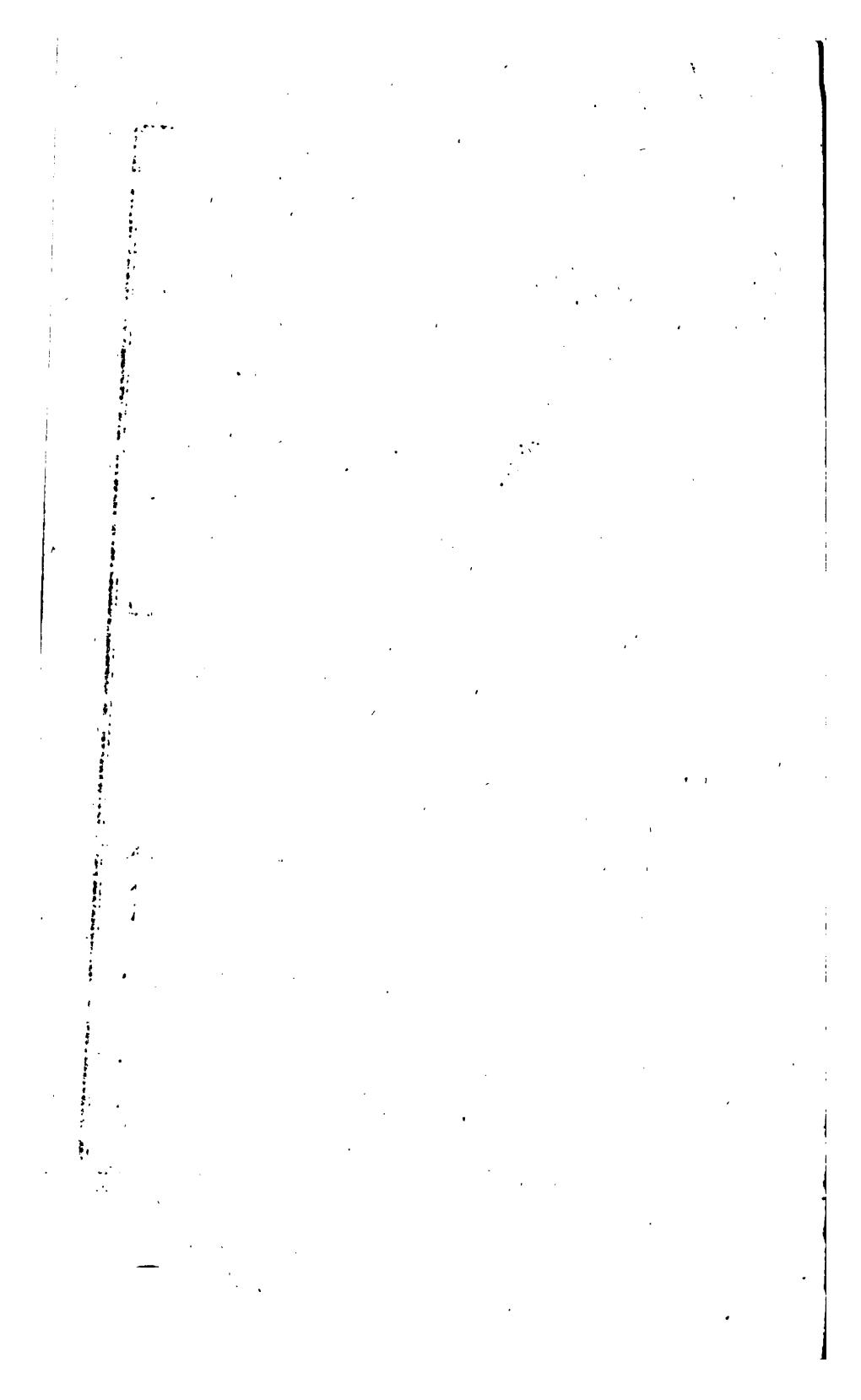
$$\times S.2T + \frac{4n^3}{6a^3 + 3ann} \times \overline{S.T}^3, \text{ because } T \text{ is nearly } = z, \text{ and } 2xs = S.2T. \text{ But since } n \text{ is very small by supposition, } z = T + \frac{nn}{4aa} \times S.2T + \frac{2n^3}{3a^3} \times$$

$$\overline{S.T}^3. \text{ Where the quantities } \frac{nn}{4aa} \times S.2T, \text{ and }$$

$$\frac{2n^3}{3a^3} \times \overline{S.T}^3 \text{ are small arches to be added to } T.$$

$$\text{Now } D = c - \frac{cc}{a} = \frac{c}{a} \times \overline{a - c}; \text{ and } D \times \overline{AO + OD} = \frac{c}{a} \times \overline{aa - cc} = \frac{cnn}{a} = nn \text{ nearly, because } c = a \text{ very near. Therefore } s.Y : \text{rad (1)} : : nn : 4aa, \text{ and } s.Y \text{ or } Y \text{ (in small arches)} = \frac{nn}{4aa}; \text{ also } V : Y :: ST : \text{rad (1)}, \text{ and } V = Y \times S.2T$$





$S.2T = \frac{nn}{4aa} \times S.2T$, which is our first term, or Fig. 24.
his first equation.

Again, $S.Z : \text{rad}(1) :: 4nD$ or $\frac{4nc}{a} \times \overline{a-c}$:
 $3AO^2$ or $3aa$, and $S.Z = \frac{4nc}{3a^3} \times \overline{a-c}$. But $\overline{a+c}$
 $\times \overline{a-c} = aa - cc = nn$, and $a - c = \frac{nn}{a+c}$
 $= \frac{nn}{2a}$ nearly, therefore $S.Z = \frac{4nc}{3a^3} \times \frac{nn}{2a} = \frac{2n^3c}{3a^4}$
 $= \frac{2n^3}{3a^3}$, nearly. Also $X : Z :: \overline{S.T^3} : \text{rad}(1)$. And
 $X = Z \times \overline{S.T^3} = \frac{2n^3}{3a^3} \times \overline{S.T^3}$, which is the second
term, or his second equation. And when T is $a.$ above 90° , then $S.2T$, and V will be negative, and
 $\angle BHP = T + X \pm V$.

He calls, Y or $\frac{nn}{4aa}$ the greatest first equation,
because it is greatest when $S.2T = 1$, and $T = 45^\circ$.
And Z or $\frac{2n^3}{3a^3}$ the greatest second equation, because
the greatest it can be is when $\overline{S.T^3} = 1$, or
 $T = 90^\circ$.

S E C T. VII.

[Pr. 33, to AB the principal semi-diameter] this
should be $\frac{1}{2}AB$.

[ib. cor. 2.] for the two first terms of the prop.
are in a ratio of equality, and so the two last.

[Pr. 34. For (by cor. 7. prop. 16.)] and cor. 6.
prop. 4.

[Pr. 35. The same things supposed] to wit, that
the space CS is as the area SDE of the circle, re-
tangled hyperbola, or parabola.

A C O M M E N T . o n

Fig. [Prop. 37, as appears by prop. 34.] and cor. 7.
25, pr. 16.

[Pr. 38, acquire the velocity CD.] by prop. 10, cor. 2. The periodical times of the ellipsis AD and AP are equal. The time of describing AP is (as APS, that is, as ADS or) as AD. Let the ellipsis AP coincide with AC, and the time of describing AC will still be as AD. Further, draw $cd \parallel$ to CD; because the time of describing the whole ellipses ADD_d, AP_p are equal; therefore in equal times they describe areas which are as the whole ellipses, that is, as CD to CP, or ADS to APS. Wherefore in the time D describe AD or D_d, P describes AP or P_p, and (when P coincides with C), C describes AC or C_c; therefore the velocity of D : to velocity of C :: is as D_d : C_c :: or as SD to CD; but SD and the velocity of D is given; therefore the velocity of C is as CD.

[Pr. 39. cor. 2, 3.] In these cor. the line PD is the space the body would ascend to, or fall from, (to acquire the velocity it is projected with) by a uniform centripetal force, according to cor. 1.

S E C T. VIII.

[Pr. 40. cor. 1.] This is evident, by supposing ITK convex towards C.

26. [ib. cor. 2.] for by fluxions, let AC = P. CD = A = x . Then DG $\propto x^{n-1}$ (by hyp. and pr. 39.) and DEFG = fluxion of the area $\propto x^{n-1}x$. And the area DCbG $\propto \frac{x^n}{n}$. And when x becomes = p, the area ACbB $\propto \frac{p^n}{n}$. Therefore ADGB $\propto \frac{p^n - x^n}{n}$. And therefore by prop. 39, the velocity in

B. I. THE PRINCIPIA.

23

in D, at the distance A or x , is $\propto \sqrt{\frac{P^n - x^n}{n}}$, or Fig. 26.

(because n is given, and $x = A$) as $\sqrt{P^n - A^n}$.

N. B. If the force is reciprocally as the distance, the curve BGh will be a rectangled hyperbola, to the asymptotes AC , Cb ; for $DG \propto \frac{1}{CD} = (\text{suppose to}) \frac{bb}{CD}$. And $DG \times CD = bb$, which is the

known property of the hyperbola. And the velocity at C will be infinite, for the area $ABbC$ is infinite. And the velocity at any place D is as the hyperbolic area $ABGD$, which may be found by Stone's Fluxions, p. 54; or by Cor. 2. Prop. 87.

B. II. my Conic Sections.

[Pr. 41.— in the Trajectories found.] One being given to find the other.

[ib. given the circle VR] that is, CV is known.

[ib. in the least given time,] these small parts of time are taken constant and invariable.

[ib. and the triangle ICK] that is, the triangle will be invariable.

[ib. and suppose the magnitude of Q,] This quantity is any constant quantity, but unknown. Suppose in some case $\sqrt{ABFD} \asymp b \times IK$, and $Z = b \times KN$. And in all cases (by pr. 40.) $\sqrt{ABFD} = b \times IK$; and in all cases Z or $\frac{Q}{A} = b \times KN$, that is in all cases $Q = b \times A \times KN$; which is plain, because Q and $A \times KN$ are constant quantities, therefore if it be once $\sqrt{ABFD} : Z :: KI : KN$, or $b \times IK : b \times KN :: KI : KN$; it will always be so.

In this prop. the line CE is indetermined; and if Q were known, the areas of the curves abz , and azx might be known to any distance CE.

A C O M M E N T o n

Fig. [ib. cor. 3.] In fig. 4. Newt. Let $CA = a$
 $CV = r$, $CD = x$, $DF = y$, $\sqrt{aa - rr} = n$.

Then the force DF being as $\frac{1}{x^3}$, the fluxion of
the area $ABFD$ is $\frac{-x}{x^3}$, and the fluent as $\frac{1}{2x^2}$,
and corrected, the fluent is $= \frac{aa - xx}{2aaxx} = \text{area}$

$ABFD$. Therefore the velocity at D is as $\frac{\sqrt{aa - xx}}{ax}$.

Then,

1. Suppose a infinite, and $x = r$, then the area
 \sqrt{ABFD} becomes $\frac{\sqrt{aa}}{ar} = \frac{1}{r}$. And since at V ,
the orbit IV is perp. to CV , therefore $IK = KN$,
and $\sqrt{ABFD} = Z = \frac{Q}{x}$, that is, $\frac{1}{r} = \frac{Q}{r}$, and
 $Q = 1$, and $Z = \frac{1}{x}$; also $\sqrt{ABFD} = \frac{\sqrt{aa - xx}}{ax}$
 $= \frac{a}{ax} = \frac{1}{x}$ at any place I , and since \sqrt{ABFD}
 $\left(\frac{1}{x}\right) : Z \left(\frac{1}{x}\right) :: IK : KN$; therefore $TK = KN$,
and $IN = a$; therefore in this case the orbit is a
circle, as VXR .

2. If a be less than infinite, and since at V , the
orbit is perp. to CV as before, therefore $IK = KN$,
and $\sqrt{ABFD} = \frac{Q}{r}$, that is, $\frac{\sqrt{aa - rr}}{ar} = \frac{Q}{r}$, or
 $\frac{n}{ar} = \frac{Q}{r}$, and $Q = \frac{n}{a}$. Whence $Z = \frac{n}{ax}$. There-
fore $Db = \frac{n}{a} \times \frac{1}{\sqrt{\frac{aa - xx}{aaxx} - \frac{nn}{aaxx}}} = \frac{n}{2a} \sqrt{\frac{aaxx}{rr - xx}} =$
 nx

$\frac{xx}{2\sqrt{rr-xx}}$. And $Dc = \frac{rr}{xx} \times Db = \frac{nrr}{2x\sqrt{rr-xx}}$, Fig. 26.

and flux of the area $VacD = \frac{nrr\dot{x}}{2x\sqrt{rr-xx}} =$ fluxi-

on of the sector VCX , and the Fl.: $\frac{nrr\dot{x}}{2x\sqrt{rr-xx}}$

= Sector VCX , which is as the angle VCX .

But, putting semiconjugate = c , $CH = Z$, the hyperbolic sector $VCR =$ Fl.: $\frac{rcz}{2\sqrt{zz-rr}}$, and CT

$= \frac{rr}{z} = s$, and $z = \frac{rr}{x}$. Then instead of z and \dot{z} , putting their values; we shall have the hyp. sector $VCR =$ Fl.: $\frac{-rrc\dot{x}}{2x\sqrt{rr-xx}} = \frac{c}{R} \times$ cir. sector

VCX . Therefore when $x = CT$, the hyp. sector $VCR : \text{cir. sector } CVX :: c : n$; that is, because the angle VCX is as the sector CVX , the angle VCX is to the hyper. sector VCR in a given ratio. And since CT or x continually decreases, as the sector VCR increases, the body P draws continually nearer the center C .

3. If the velocity be greater than falling from an infinite height; the flux area $ABFD = \frac{+x}{x^3}$ and the fluent $\frac{-1}{2xx}$. But at first the area

(suppose) $= \frac{bb}{2}$, and $x = a$ an infinite line; there-

fore $ABFD - \frac{bb}{2} = \frac{1}{2aa} - \frac{1}{2xx}$ and $ABFD =$

$\frac{bb}{2} + \frac{xx-aa}{2aaxx}$. Therefore \sqrt{ABFD} is as

$\sqrt{\frac{bbaaxx+xx-aa}{aaxx}} = (\text{by substitution}) \sqrt{\frac{sxx-aa}{aaxx}}$.

But

A C O M M E N T O N

Fig. But in V, $x = r$, and $\sqrt{ABFD} = \sqrt{\frac{srr-aa}{aarr}} =$

$$\frac{Q}{r}, \text{ and } Q = \frac{\sqrt{srr-aa}}{a}, \text{ and } Z = \frac{Q}{x} = \frac{\sqrt{srr-aa}}{ax}.$$

$$\text{Then } Db = \frac{\sqrt{srr-aa}}{\frac{2a}{\sqrt{aaxx}} \sqrt{\frac{sxx-aa}{aaxx} - \frac{srr-aa}{aaxx}}} = \sqrt{\frac{srr-aa}{s}}$$

$$\times \frac{x}{2\sqrt{xx-rr}} = (\text{by substitution}) \frac{Ax}{2\sqrt{xx-rr}}, \text{ and}$$

$$Dc = \frac{rr}{xx} \times Db = \frac{Arr}{2x\sqrt{xx-rr}}; \text{ therefore}$$

$\frac{Arrx}{2x\sqrt{xx-rr}}$ = flux. area VacD, or of VCX. And

the Fl. : $\frac{Arrx}{2x\sqrt{xx-rr}}$ = sector VCX, which is as the angle VCX.

27. But in the ellipsis VRS, let CH = z, semiconjugate = c. Then the elliptic sector VCR = Fl. :

$$\frac{rcz}{2\sqrt{rr-zz}}, \text{ and } CT = \frac{rr}{z} = x, \text{ and } z = \frac{rr}{x},$$

and $\dot{z} = \frac{-rr\dot{x}}{xx}$. Then putting for z and \dot{z} their

values, and the elip. sector VCR = Fl. : $\frac{-rr\dot{x}}{2x\sqrt{xx-rr}}$

$= \frac{c}{A} \times \text{cir. sector VCX}$; therefore when $x=CT$,

the ellip. sector VCR : cir. sector VCX :: c : A; that is, because the angle VCX is as the sector VCX; the angle VCX is to the elliptic sector VCR in a given ratio. And since CT or x continually increases, as the sector VCR increases, the body P goes continually further and further from the center C.

[ib. and the Centripetal Force becoming centrifugal] for then the curve VPQ will turn upwards;

wards; and the law of the force being the same, Fig. it will be constructed the same way by the elliptic sectors; taking the point A between C and V. And here the velocity will increase, as it recedes from the center C. But it can never revolve round this center.

S E C T. IX.

[Pr. 44. or in antecedentia with a celerity] for then the line mnC falls beyond s , sr being $= rk$, and $sCk = 2rCk$; and consequently the point m falls without the circle.

[ib. and with a less force] if the orbit moves slower in antecedentia, than with twice the celerity of CP in consequentia; for then the point m falls within the circle, between r and s .

The meaning of this Prop. is this, that the difference of the forces at different distances from the center, are reciprocally as the cubes of the distances (which forces are requisite to make the body move in a quiescent or revolving orbit; and the distances to be the same in both.)

[ib. cor. 1.] For mn represents the difference of the forces by which p revolves to n , or P to K in equal times; and the versed sine of RK represents the force whereby a body moves from R to K in the circle, in the same time.

[ib. cor. 2. as half the latus rectum] for these forces are as bn or ar to bp or as . But $ar = \frac{nr^2}{rD} = \frac{ba^2}{aD}$.

And $as D : sp^2 :: aD : 2R$; and $as D = \frac{sp^2 \times aD}{2R}$

$= \frac{ab^2 \times aD}{2R}$; and $as = \frac{ab^2 \times aD}{sD \times 2R} = \frac{ab^2}{2R}$. Whence

these forces are as $\frac{ba^2}{aD}$ to $\frac{ba^2}{2R}$, or as R to aC or CV in fig. 2. pl. 18.

[ib.

28.

A C O M M E N T o n

Fig. [ib. cor. 3.] For let the force in the immovable ellipsis be $\frac{FFA}{T^3}$, and in V it will be $\frac{FFT}{T^3}$ or $\frac{FF}{TT}$. And the force by which a body may revolve in a circle at the distance CV is $\frac{FFR}{T^3}$. And the difference of the forces in V (of revolving in the moveable and immovable ellipsis) is $\frac{GGR-FFR}{T^3}$, and in every altitude A is $\frac{GGR-FFR}{A^3}$; and the force in the moveable ellipsis will be $\frac{FFA}{T^3} + \frac{RGG-RFF}{A^3}$.

[ib. cor. 4.] After the same manner as in the two foregoing Corollaries; because when the velocity is given, the force \propto curvature or reciprocally \propto radius of curvature; therefore $T : R :: \frac{VFF}{TT}$: $\frac{RVFF}{T^3}$, which is the force wherewith a body may revolve in a circle at the distance CV. And by cor. i. $FF : GG - FF :: \frac{RVFF}{T^3} : \frac{RVGG-RVFF}{T^3}$, which is the difference of the forces (in the moveable and immovable orbits) in the vertex, V. And by the prop. $\frac{I}{T^3} : \frac{I}{A^3} :: \frac{RVGG-RVFF}{T^3} : \frac{RVGG-RVFF}{A^3}$, which is the difference of forces in every altitude A; and therefore

fore the force in the moveable trajectory Vpk will 27.
be $X + \frac{RVGG - RVFF}{A^3}$.

[ib. cor. 6.] In this cor. $X = 0$, and $RVGG - RVFF$ is compounded of all given quantities, and the force in Vpk will then be $\propto \frac{1}{A^3}$. This also appears from the Prop. for the difference of the forces in P and p is as $\frac{1}{Cp^3}$, but the force in one of them, viz. at P in the line VP is 0 ; and therefore the other force in Vpk is the difference of the forces, and is as $\frac{1}{Cp^3}$. Also this curve is the same with that in cor. 3. Pr. 41. for let vad be a circle; and 28. by this cor. 6. $\angle vcp \propto \angle vcx \propto$ arch $va \propto$ area $vca \propto$ area vcb , which is the construction in cor. 3. Pr. 41; also $cp = cx$ or cx , because vz , ax , and bx are tangents, which is also the same construction as in cor. 3. Pr. 41.

[Pr. 45, but orbits acquire the same figure] Let 29. the small parts of the curve $df, fb, bp, &c.$ be described by a body A in indefinitely small given parts of time. And let another body B go from D in the same direction de , and with a velocity which is to the velocity of the former in the sub-duplicate ratio of the centripetal force of B to that of A ; and let it arrive at s in the aforesaid small given part of time. Now since $ef : rs : : \text{force of } A : \text{force of } B : : \text{velocity}^2 \text{ of } A : \text{vel.}^2 \text{ of } B : : de : dr^2 : : \text{square of the time of } B\text{'s moving thro' } de : \text{square of the time of } B\text{'s moving through } dr$. Therefore, by Lem. 10, the body B will pass thro' f . And since rs is every where as rd^2 , it is evident the same curve dsf will be described by both bodies, and which $defg$ touches in d, f . Also the velocity of A in $f : \text{vel. of } A \text{ in } d : : \text{as perpendicular from } C \text{ or } de :$

Fig. de : to \perp on fg : : vel. of B in f : vel. of B in d .
 29. And vel. of A in f : velocity of B in f : : vel. A in d : vel. of B in D : : $de : dr$; that is in a given ratio; that is fg : to fk , and bn to bl , and px to pt are in a given ratio; and therefore when A is arrived at g , B would be at k , and since $gb : ki : : gf : kf^2 : : ed^2 : rd^2 : : ef : rs$; or $gb : ef : : ki : rs$; that is, the centripetal of A to that of B at equal distances being in a given ratio, B will pass thro' i , and consequently thro' b . After the same manner it may be proved, that the body B will pass thro' the points m , p , u , y , and describe the same curve with the body A.

Further, if another body D move thro' $Br \parallel$ to de , and with a velocity which is to the velocity of A as CB to Cd , and be acted upon by a force which in the points β , δ , z , &c. is to the force in the points d , f , b , &c. respectively, as $C\beta$ to Cd , $C\delta$ to Cf , Cz to Cb , &c. Then I say the curve described by the body D, viz. $\beta\delta z\varphi\omega$ will be similar to the curve $dfbpy$; for in the time that A would arrive at e , D would arrive at y ; and because $Cy : Ce :: C\beta : Cd : : \text{force at } \beta : \text{force at } d :: \gamma\delta : ef : : Cr - \gamma\delta$ or $C\delta : Ce - ef$ or Cf ; and $C\beta : Cd : Cd : Cf$; $\gamma\delta$ is every where as $\gamma\beta^2$ in all the points between β and γ (by Lem. 10.) it is manifest the fig. $C\beta\delta$ is similar to the fig. $C\delta f$; and the tangent $\delta e \parallel$ to fg ; and because the areas $Cgf : Cfd :: Cey : Cd\beta$; wherefore when A comes to g , D comes to e ; but $gb : ez :: Cf : Cd :: Cg : Ce :: Cb : Cz$, as before; and $Cf : Cb :: Cd : Cz$, whence the fig. $C\delta z$ is similar to Cfb ; and after the same manner it will be proved, that the nascent figures $Cz\varphi$, $C\varphi\omega$, are similar to Cbp , Cpy ; and therefore the whole figure $\beta\delta z\varphi\omega$ is similar to the figure $dfbpy$.

Or universally, if the orbits $\beta\delta\varphi$, and dfp are similar. $C\beta \propto \gamma\delta \propto \text{force} \times \text{time}^2$ of describing

bing $\beta \propto$ force $\times \frac{\beta^3}{\text{velocity}^2}$ force $\times \frac{C\beta^2}{\text{velocity}^2}$. Fig. 30.

Therefore velocity 2 \propto force \times distance $C\beta$. Whence if $C\beta$ be given, velocity $\propto \sqrt{\text{force}}$; and if velocity $\propto C\beta$, force $\propto C\beta$ also, which agrees with what went before. If the force be given, velocity

$\propto \sqrt{C\beta}$. If velocity be given, force $\propto \frac{1}{C\beta}$.

[ib. Exam. 1. l. 3.] here $\frac{T^3 - 3TTx + 3TX^2 - X^3}{A^3}$

is twice repeated in the English, which is wrong.

[ib. By this collation of the terms] for since the same figure will be described by making the centripetal force proportional at equal distances (altering the velocity in the subduplicate ratio of the force); and since, when R is nearly $= T$; and x very small, the centripetal force in the revolving ellipsis will be as $RGG - FFx$, and in this orbit (because in both, the denominator A^3 is the same) is as $T^3 - 3TTx$, therefore these forces ought to be proportional at all distances, viz. when $x = o$, and any other indeterminate distance A or $T - x$; wherefore (in these two cases) $RGG : T^3 :: RGG - FFx : T^3 - 3TTx ::$ (and by division) $FFx : TTx :: FF : TT$, which is the construction of the problem. After the same manner in example 2d, the centripetal force in the revolving ellipsis, and this new orbit, will be as $RGG - FFx$, and $T^n - nT^{n-1}x$, and to have the orbits similar, putting $x = o$, $RGG : T^n ::$ (putting $T - x$ for any indeterminate distance) $RGG - FFx : T^n - nT^{n-1}x ::$ (by division) $FFx : nT^{n-1}x :: FF : nT^{n-1}$. After the same manner in Examp. 3, it will be $RGG : bT^n + cT^n ::$ ($RGG - FFx : bT^n + cT^n - mbT^{n-1}x - ncT^{n-1}x$) :: $FFx : mbT^{n-1}x + ncT^{n-1}x$. Or $GG : bT^{n-1} + cT^{n-1} ::$

..

Fig. :: $FF : mbT^{n-1} + ncT^{n-1}$. The quantity $\frac{FF}{AA}$
 $+ \frac{RGG - RFF}{A^3}$ (in cor. 2. pr. 44.) is universally
as the centripetal force, whether the apsides move
backwards or forwards, for $G : F :: VCp : VCP ::$
(p. 180.) $pCn : pCk$. And the force in the revolv-
ing ellipsis is greater than in the immovable one,
when n (fig. 2.) is without the line ks ; and in that
case (F is less than G , and) $+ \frac{RGG - RFF}{A^3}$ will
be affirmative. But the force is less when n falls
between k and s , for then (F is greater than G ,
and) $+ \frac{RGG - RFF}{A^3}$ is negative, as it ought to be.

S E C T. X.

[Pr. 50. it is evident from the construction]
Since VP touches the curve in P, and PB is \perp thereto, therefore B is the point of contact of the circle AD, and the wheel or generating circle, and therefore $BV = AO$. Also since WT is \perp to TV, therefore V is the point of contact of the generating circle (whose diameter is VW) and OS, whence $VW = OR$. The figures are similar, because their axes are as the radii of the spheres.

[Pr. 52. hence since in unequal—] suppose TR
30. to be a smaller oscillation, to find the velocity in I ;
take $st : SR :: TI : TR$; then velocity at R in the
arc TR : velocity at R in SR :: TR : SR ; and
velocity at I in TR : velocity at R in TR ::
(Prop. 51.) velocity at t in SR : velocity at R in
SR :: $\sqrt{SR^2 - tR^2} : SR$. And ex equo velocity at
I in TR : velocity at R in SR :: $TR\sqrt{SR^2 - tR^2} : SR$
 $SR^2 :: \frac{TR}{SR}\sqrt{SR^2 - tR^2} : SR ::$ (because SR :
 $TR :: tR : IR$ as) $\sqrt{TR^2 - IR^2} : SR$.

[ib.

[ib. There are obtained from the times given] Fig. for in two unequal arches, two corresponding parts are described in equal times (by Prop. 51.) therefore the velocities in these points will be as the entire arcs; and therefore both the velocities and arches will be known.

[ib. And if the absolute force of any globe] that is, the force at a given distance be called V ; then the force at the distance CO is $CO \times V$. But in a given time, HY is as the force; whence $CO \times V$ is as HY , and therefore HY is described in a given time.

[ib. cor. 1. For this time] for then $AR = AC$, and V is given; therefore, this time : time of $\frac{1}{2}$ oscillation :: $\sqrt{\frac{AC}{AC}} : \sqrt{\frac{AR}{AC}} :: 1 : \sqrt{\frac{AR}{AC}}$.

[ib. cor. 2. But in that case] for (in fig. 2. pl. 19.) by cor. 1, 2, pr. 49. $AS : Bv :: PS : PV :: 2CE : CB$ (in this case as) :: 2 : 1 :: (and by division) $AP : BV - PV$; whence $AP = 2BV - 2PV$; but verfed sine of $\frac{1}{2}PB = \frac{BV - PV}{2}$, for cosine = $\frac{1}{2}PV$;

and $\frac{1}{2}VB - \frac{1}{2}VP =$ verfed sine.

[ib. as M. Huygens] all this is demonstrated in Keil's Philosophy. See my large book of Mechanics, Prop. 40. cor. 4.

[Pr. 56, let the projection] T is projected into P and t into p .

[ib. as also its position] The greater axis $2Tt$ is perpendicular to PO .

[ib. and since the area POp] for let $\frac{\text{area } OPp}{OP} = sp$, 31.
which is given (sp being \perp to PO); and $PB^2 : Pr^2 :: PB^2 - SP^2 : PS^2$; whence Ps is given, and thence SO , the angle POp , Op and Pp , the point p , and angle OPp .

S E C T. XI.

[Pr. 57, and about each other] considering either body as at rest.

A C O M M E N T •

Fig. [Pr. 58, by the same forces there may be described, &c.] that is, by making the velocity to the former velocity as (\sqrt{sp} to \sqrt{CP} , that is as) $\sqrt{P+S}$ to \sqrt{S} , as appears by the Prop.

[Pr. 60, in a ratio sesquiplicate] it should be sub-sesquipuplicate.

[ib. of the other ellipsis.] As $S^{\frac{1}{2}} : S + P^{\frac{1}{2}}$. Let $S^{\frac{1}{2}} : a : e : S + P^{\frac{1}{2}}$ be \therefore . Then $S^{\frac{1}{2}} : S + P^{\frac{1}{2}} :: (S : a^2 : :) e^2 : S + P$. But (since $S + P : e^2 :: a^2 : s$) e^2 is the first of two mean proportionals between $S + P$ and S .

[Pr. 61.] Note, if a body were placed in the center of gravity, and whose force would be sufficient to cause one of the bodies to revolve around it, and to describe the same figure; yet it would not cause the other body (if it were unequal) to describe its figure, except in that law of centripetal force, which is as the distance where the periodic times are all equal. For, to preserve the same motion as before, either body must be attracted to the body in the center of gravity, with a force which is as the other body, or as its own distance, which is the case of one body attracting another; and holds only (in the case of their being attracted by a third body in the center of gravity) in the law of centripetal force before-mentioned. And therefore in

[Pr. 62.] The foregoing note is to be observed.

[Pr. 64. This would be the case] For this only causes the bodies T, L, to revolve more swiftly round their center of gravity D, but affects not the other bodies.

[ib. with equal periodical times.] By the foregoing (in the three bodies S, T, L) the body S describes an ellipsis round C; and, by considering the center of gravity of S, T, as describing an ellipsis (as before of T, L,) it will appear the same way that the body L describes an ellipsis round C; and as the center of gravity of S, L, describes an ellipsis,

ellipsis, so T describes an ellipsis also, round C. Fig. Further, from what went before, the centripetal force of S towards C, is as $T + L \times SC + CD$, or $T + L + S \times SC$. Also the centripetal force of T towards D, is as $L \times TD + DL + S \times TD$ (which latter part arises from the resolution of the force ST into SD, DT, and DT acts towards D), or as $T + L + S \times TD$. Now by cor. 2, 8. pr. 4. The periodic times are in the subduplicate ratio of the radii directly, and the subduplicate ratio of the forces inversely: therefore the periodic time of S, round C : periodic time of T, round D :

$$\sqrt{\frac{SC}{SC \times T + L + S}} : \sqrt{\frac{TD}{TD \times T + L + S}}; \text{ and are therefore equal; so that after one revolution, the bodies all return to their first places.}$$

After the same manner the point C, and a fourth body V, as also the bodies S, T, L, will describe ellipsis round their common center of gravity B, for any one, and the center of gravity of the other three will describe ellipses; and the case is the same if there were more bodies. Also, as before, and by a like resolution of forces, since the forces VS, VT, VL, are resolved into VC, CS; VC, CT; VC, CL; the former acting in the direction of CV towards V, the latter to C their center of gravity; and are as the distances of the bodies therefrom, as before; they will therefore still move round their center of gravity C as before, but swifter, altho' all the four revolve round B. Also periodic time of V round B : periodic time of S round C :

$$\sqrt{\frac{BV}{S + T + L \times CB + BV}} : \sqrt{\frac{SC}{T + L \times SC + V \times SC}} \\ :: (\text{that is, as}) \sqrt{\frac{BV}{T + L + S + V \times BV}} : \\ \sqrt{\frac{SC}{T + L + S + V \times SC}}; \text{ and are therefore equal;}$$

Fig. and if more bodies were added, all the bodies would perform their revolutions in equal periodic times. Q. E. D.

[Pr. 66. cor. 2. Such is the force NM.] This force always acts from P in a direction parallel to TS, and from M towards T.

[ib. cor. 6. the periodic time will be increased,] for then the periodic time is $\propto \frac{\text{Rad}^{\frac{3}{2}}}{\sqrt{\text{force}}}$, which is a greater ratio than $\text{Rad}^{\frac{3}{2}}$, because the force is diminished.

[ib. cor. 7. the upper apsis to go backwards] for $\frac{n^2}{m^2} - 3 > -2$. And $\frac{n^2}{m^2} > 1$; therefore n is $>$ than m . Also put D = distance (r = a given quantity,) and the force is $\left(\frac{D^2 + r^2}{D^2} \right)$, or rather) as

$\frac{1}{D^2 + \frac{1}{D}}$, and when the distance increases, the de-

crease of the force is $D^2 + \frac{1}{D}$, which is less than

$D^2 + \frac{D^2}{D}$, which is the decrease according to the duplicate ratio. Also at the conjunction, &c. the force of P towards T is, the centripetal force of T + LM — TM, or the centripetal force of T — KL. Also the decrease of the force is as $d^2 - \frac{1}{d}$, which is greater than $d^2 - \frac{d^2}{d}$, or the decrease according to the duplicate ratio. And $\frac{n^2}{m^2} - 3 < -2$; and $n < m$.

[ib. cor. 7. The truth of this] for then the further P recedes from T the more it is attracted towards the bodies S, S, and therefore less towards T, than it would otherwise be.

[ib.

[ib. cor. 8. when the apsides are in the syzyges] Fig. for then NM is greater, and LM less, than before.

[ib. cor. 9. Now therefore —] For the ratio of KL to LM is least when the apsides are in the quadratures, and greatest in the syzyges.

[ib. For the forces LM] as is shewn in cor. 7.

[ib. cor. 10. from the syzyges to the quadratures] for after the syzyges, the body P by the action of S, is made to move in lines which successively cut the plane TPS at greater and greater angles. And for the same reason at the 90° from the octants (between C and A), the body is drawn into lines (or little planes) which cut TPS at greater and greater angles, which before 90° cut it at less and less.

[ib. and by a like reasoning] for the inclination 32. is increased from μ to D, diminished from D to S, increased from S to C, diminished from C to μ ; as is plain by supposing T and any point p wherein the body is, to be joined; and the plane pTt to revolve round pT , till it pass thro' the new place of the body p , (which it acquires either above or below the plane pTt ,) by the (action of S upon it, or the) force NM; and then it will appear, how that new acquired plane cuts the plane SCT, whether in a greater or lesser angle.

[ib. cor. 11.—from the former plane CD] Since the inclination of the orbit is diminished from C to A, the intersection with TSE will move from C to B, and from D to A; and seeing that inclination is increased from A to D, the intersection (or nodes) will still move towards B and towards A.

[ib. being always either retrograde] that is, considering a whole revolution; for in some points they go forward.

[ib. cor. 13. And since the causes and proportions] to one another, and to the force of S, &c.

A C O M M E N T O N

Fig. [ib. cor. 14. But since the forces] For $SK : LM$
 $::$ accelerating force of T towards S : perturbating
 force of S . And therefore the perturbating force
 of $S = \frac{LM \times \text{accelerating force of } T \text{ towards } S}{SK}$.

But since $SK = ST$ nearly, and LM (in its mean
 quantity) $= PT$. And the accelerating force of T
 towards S , is as the body S directly, and ST^3 re-
 ciprocally; therefore the perturbating forces LM ,
 $NM \propto \frac{PT \times \text{body } S}{ST^3}$, that is, \propto

$$\frac{PT \times \text{bodies}}{ST^3}$$

Or $\propto PT \times \text{den-}$
 \square periodic time of T round S
 \times density of $S \times \text{cub. apparent diameter of } S$. (for the
 apparent diameter is as the real diameter directly,
 and the distance reciprocally)

[ib. cor. 16, square of the periodical time of the
 body P conjunctly.] This holds as true in the sy-
 nodic time of the body P , (since the action of the
 forces that cause these errors begin and end at the
 quadratures, which comes to the same, as if they
 begun and ended at the syzyges) see what follows.

[ib. and hence the angular errors] these in one
 revolution are as the linear errors directly, and the
 radius or distance reciprocally, that is, as the forces
 and square of the time of revolution of P , direct-
 ly, and the distance TP reciprocally; that is (be-
 cause the force $\propto TP$) as square of the time of
 revolution.

[ib Let these ratio's] The angular errors of P
 observed from T are as $\frac{\text{forces} \times \text{time}^2}{TP}$, that is (by

cor. 14.) as $\frac{PT \times \text{body } S \times \text{time}^2}{PT \times \square T \text{ periodic time}}$, that is, in
 the time of one revolution of P (and if S be gi-
 ven), as $\frac{\square \text{time of } P \text{ s revolution}}{\square \text{periodic time of } T}$.

[ib.

[ib. both these motions will be as the periodical Fig. time of the body P, directly,] for the angular motion or velocity, or the mean angular errors, are as the sum of all the angular errors in any time directly, and the time reciprocally; that is, (in the periodic time) as

$$\text{body } S \times \square \text{ time of P's revolution}$$

time of P's revolution $\times \square$ periodic time of T, that is, as body S \times time of P's revolution directly, and the square of the periodic time of T, reciprocally.

[Cor. 20. And thence the greatest height of the water] This is explained by Worster in his (Experimental) Philosophy, page 72. Also see my Geography, Prop. 12. Sect. I.

S E C T. XII.

[Pr. 79. As the lineola Dd,] when the radius of the sphere is given; and when the radius is not given, as Dd and the radius conjunctly; therefore AS, and also PS or PE, may be of any magnitude,

[ib. the sum of all the rectangles PD \times Dd] that is, as $\frac{PF + PD}{2} \times DF$, or $\frac{PF + PD}{2} \times \overline{PF - PD}$, or $\frac{PF^2 - PD^2}{2}$.

[Pr. 80. The whole force of the sphere will be as the whole area ANB.] Also it is manifest, that the attraction or force of any part of the sphere FEB, is as the correspondent part of the area DNB.

[Pr. 81. Ex. 1.] The area SL into AB = SL \times AB, because both SL and AB are given. And the area LD into AB is a trapezoid, whose base is AB, and the two parallel sides LA and LB; for as the point D removes nearer A, the ordinate LD decreases

Fig. decreases (from B where it is LB) to A, (where it
 32. is LA); and that area is $\frac{LA + LB}{2} \times AB$, or

$$\frac{LA + LB}{2} \times LB - LA, \text{ that is, } \frac{LB^2 - LA^2}{2};$$

$$\text{or it is } \frac{2LA + AB}{2} \times AB, \text{ that is } LS \times AB. \text{ And}$$

the ordinate $\frac{ALB}{LD}$, is the ordinate of an hyperbola
 between the assymptotes, whose center is L, and one
 assymptote LB; for ALB is a given rectangle, and
 the nature of the hyperbola is that $\frac{ALB}{LD}$ (LD the

part of the assymptote being taken as the axis) is
 as the ordinate erected on D; whence, the reason
 of the construction will be evident; for $aAbb$ is
 the aforesaid trapezoid, by construction; also in the
 hyperbola ab , the rectangle $LB \times Bb$ is given, but
 that (by construction) is = the rectangle ALB ;
 when the hyperbolic area $aAbb$ is that described

the ordinate $\frac{ALB}{LD}$. And the difference of

and the trapezoid (which is the difference
 two first areas) is the area aba required.

[ib. Ex. 2. the third $\frac{ALB \times SI}{2LD^2}$) this is
 calculated by fluxions, LD is a flowing qua-

the fluxion of the area is $\frac{ALB \times SI}{2} \times LD^{-2}$

And the whole area is $\frac{ALB \times SI}{2} \times LD^{-1}$,

$\frac{ALB \times SI}{2LD}$, or (when LD = LB) $\frac{ALB \times SI}{2LB}$:

this take the area (when LD = LA,

B. I. THE PRINCIPIA.

41

ALB × SI, and the difference is the area sought Fig.
 $\frac{1}{2}LA$

$$\frac{ALB \times SI}{2LA} - \frac{ALB \times SI}{2LB}$$

[ib. Ex. 3.] The fluxion of the first area is
 $\frac{SI^2 \times SL}{\sqrt{2SI}} \times LD - \frac{1}{2} \times LD$, and the whole area is
 $\frac{2SI^2 \times SL \times LD - \frac{1}{2}}{\sqrt{2SI}}$, or $\frac{2SI^2 \times SL}{\sqrt{2SI} \times \sqrt{LB}}$, and that
part of the area on AB is $\frac{2SI^2 \times SL}{\sqrt{2SI}} \times$

$\frac{I}{\sqrt{LA}} - \frac{I}{\sqrt{LB}}$. The fluxion of the second area is
 $\frac{SI^2}{2\sqrt{2SI}} \times LD - \frac{1}{2} \times LD$; and the whole area
 $\frac{SI^2}{\sqrt{2SI}} \times LD - \frac{1}{2}$, and the area on AB is $\frac{SI^2}{\sqrt{2SI}} \times$
 $\sqrt{LB} - \sqrt{LA}$. Also the fluxion of the 3d area is
 $\frac{SI^2 \times ALB}{2\sqrt{2SI}} \times LD - \frac{1}{2} \times LD$; and the whole area
 $\frac{SI^2 \times ALB}{3\sqrt{2SI}} \times LD - \frac{1}{2}$; and the area on AB is
 $\frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \frac{I}{\sqrt{LA^3}} - \frac{I}{\sqrt{LB^3}}$.

[ib. And those after due reduction] LA, LI,
LB are \vdots ; for SP = $\frac{SA^2}{SI}$. PI = $\frac{SA^2}{SI} - SI$.

LI = $\frac{SA^2 - SI^2}{2SI}$. And LA = PS - AS - LI =

$\frac{SA^2 + SI^2}{2SI} - SA$. And LB = $\frac{SA^2 + SI^2}{2SI} + SA$.

But $\frac{SA^2 + SI^2}{2SI} - SA : \frac{SA^2 - SI^2}{2SI} : \frac{SA^2 + SI^2}{2SI} + SA$,

A C O M M E N T O N

Fig. SA, are $\frac{1}{2}$, as will appear by multiplying them.

Hence $\sqrt{LB} - \sqrt{LA} = \sqrt{2SI}$. For $2\sqrt{LB \times LA} = 2SI = 2LS - 2SI = LB + LA - 2SI$. And $LB + LA - 2\sqrt{LB \times LA} = 2SI$. And (by Evolution) $\sqrt{LB} - \sqrt{LA} = \sqrt{2SI}$. Therefore the first area $(= \frac{2SI^2 \times SL}{\sqrt{2SI}} \times \frac{\sqrt{LB} - \sqrt{LA}}{\sqrt{LB \times LA}}) = \frac{2SI^2 \times SL}{LI}$.

And the second area $= SI^2$. And $\sqrt{LB^3} - \sqrt{LA^3} = LB + LA \times \sqrt{LB} - \sqrt{LA} + \sqrt{LB} \times \sqrt{ALB} - \sqrt{AL} \times \sqrt{ALB} = 2LS \times \sqrt{2SI} + \sqrt{ALB} \times \sqrt{2SI} = 2LS + LI \times \sqrt{2SI} = \frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \sqrt{2SI}$. Whence the third area $= \frac{SI^2 \times ALB}{3\sqrt{2SI}} \times \sqrt{LB^3} - \sqrt{LA^3} = \frac{SI^2 \times 3CI + 2SI}{3\sqrt{ALB}} = SI^2 + \frac{2SI^3}{3LI}$.

[Pr. 83. let us suppose] it should be, let that superficies be not a purely, &c. as it is in the original.

S E C T. XIII.

[Pr. 90. cor. 1, 2.] for the fluxion of the area is as $D^{1-\frac{1}{n}}$, and the area as $\frac{D^{1-\frac{1}{n}}}{1-n}$ or as $\frac{I}{D^{1-\frac{1}{n}}}$; and the area ALIH, as $\frac{I}{PA^{n-1}} - \frac{I}{PH^{n-1}}$.

[Pr. 91. cor. 1. And the other part $\frac{PF}{PR}$] for the fluxion of that area is $= \frac{PF}{PR} \times P\dot{F} = \frac{PF}{\sqrt{RF^2 + PF^2}} \times P\dot{F} = \frac{1}{RF^2 - PF^2} \times PF \times P\dot{F}$. And the area $= \sqrt{RF^2 + PF^2} = PR$. And (the area on PB

PB is PE—AD; and on PA, PD—AD. And Fig. their difference) PE—PD, is the area on AB.

[Pr. 91. cor. 2.] This is calculated in the Appendix, by the help of the Quadratures of Curves, which see in Harris's Lex. vol. 2.

And the curve KRM is a conic section; for the indetermined quantities rise only to two dimensions. Let SA = r . SC = c . BE = π . PB = b .

$$\overline{ER^2} = \overline{PD^2} = \overline{PE^2} + \overline{ED^2} = \overline{b - u^2} + \frac{cc}{rr} \times$$

$$\overline{2ru - uu} = \overline{bb} + \frac{2cc}{r} - \overline{2b \times u} + 1 - \frac{cc}{rr} \times \overline{uu}.$$

[Sch. Pr. 93, and I suppose the force proportional.] 33.
For consider A, B, as flowing quantities, and let

\dot{A} be given = o . Now since $A^{\frac{m}{n}} = B$; therefore when A becomes $A + \dot{A} = A + o$, B becomes

$$B + \dot{B}, \text{ and then } B + \dot{B} = \overline{A + o}^{\frac{m}{n}} = \overline{A}^{\frac{m}{n}} + \frac{m}{n}$$

$\rho A^{\frac{m-n}{n}} + \frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$ + &c. Now if any quantity $A + o^2$ (where o is infinitely small) be involved; the first, second, third, &c. term, will be respectively as the flowing quantity ($A^{\frac{m}{n}}$), the first, second, &c. fluxion of that quantity, as is demonstrated in the quadrature of curves, Sch. to Pr. 11. (for which see Harris's Lexicon.) Therefore $\frac{m^2 - mn}{2n^2} ooA^{\frac{m-2n}{n}}$ is as the second fluxion of

$A^{\frac{m}{n}}$. Now if a (centrifugal) force act from the line CD in the direction B, and a body move in the curve CZ, the fluxion of A will continue the same always, and the force acting upon the body will be every where as the second fluxion of B, or

Fig. of $A^{\frac{m}{n}}$ (its equal), and therefore as $\frac{m^2 - mn}{2n^2}$
 $opA^{\frac{m-2n}{n}}$, or (because o is given,) as $\frac{m^2 - mn}{n^2} A^{\frac{m-2n}{n}}$,
or as $\frac{m^2 - mn}{n^2} B^{\frac{m-2n}{m}}$. Or $\frac{mn - m^2}{n^2} B^{\frac{m-2n}{m}}$ is as the
centripetal force. This may be easier shewn by flux-
ions only, by finding the 2d fluxion of (B or) $A^{\frac{m}{n}}$.

S E C T. XIV.

34. [Pr. 94. equal to the square of HM] for imagine.
35. a diameter drawn through H, and an ordinate to it
through I, the abscissa will be $= MI$, and the ordi-
nate $= MH$. Also completing the figure, the
tangent IP cuts from the diameter $H\alpha$, $Hp = Hu$
 $= MI$; therefore the triangles HLP , MLI are
equal and similar, and $HL = LM$. And the latus
rectum to $H\alpha$ is always the same, whatever the an-
gle of incidence be, if the velocity be given; for
the line Hd , that the body will describe in a given
time, is given; and the line dg , that the body ap-
proaches the plane in a given time is given; and
therefore $\frac{Hd^2}{Hz}$, or $\frac{zg}{zH}$, or the latus rectum, is

given.

[Pr. 96. Sch. of the secants] of the angles be-
tween the line of incidence and the plane.

[Pr. 97, cor. 2.] The curve lines CP, CQ (be-
ing every where perpendicular to AP, DK,) are
composed of arches of circles; therefore PD is the
increment of AP or AC, and QD the decrement
of QK or CK; and therefore these increments are
as the sines of incidence and emergence. And *& contra*, if PD, QD are as the sines of incidence
and emergence, a body moving in the line PD,
shall emerge in the line QDK, by this Prop.

[Pr.

[Pr. 98. and therefore QS to be always equal to Fig. CE ;] For since $qqqq$ and $ssss$ are always perpendicular to qs , therefore supposing QS and $1q.s$ to intersect in K ; then $KQ - KS = K.1q - KS$, that is $qs = 1q.s$. Also let $1q.K$ and $2q.s$ intersect in T, then $1q.s = 1q.T - ST = 2T - sT = 2q.s$. Also $2q.s = 2q.V - SV = 3q.V - SV = 3q.S$; and so on, 'till at last qs , coinciding with CE, will be equal thereto. Therefore QS is every where equal to CE.

[ib. Sch] See the Author's Optics, where all these things are shewn.

B O O K II.

S E C T. I.

[Pr. 2. cor. For if that area] by Marq. Hospital's Conic Sect. art. 219. And my Conic Sect. Prop. 86, Hyperbola, cor. 6.

[ib. of the right line AC] by the converse of Lem. 1; but the contrary does not hold, that is, if any area ABGD be taken for the time, that DC shall represent the velocity, or AD the space.

[Pr. 3. And the resistance] The force of gravity may be compared with the resistance of the medium; for they may both be considered as uniform pressure.

[ib. or AC to $\frac{1}{2}AK$] For suppose a tangent to be drawn to the point B, then $Kq : (\text{subtangent } \equiv) AC :: kq : Bk \text{ or } AK$. And in general in any area, as $MstN$; it will be, $AC : \frac{1}{2}MN + NC :: (\text{by the nature of the hyperbola}) \frac{Ms + Nt}{2} : AB \text{ or } Mm$.

And by division $AC : AM + \frac{1}{2}MN :: \frac{Ms + Nt}{2}$

$:: \frac{ms + nt}{2} :: \text{area } sMNt : \text{area } smnt$. But $AG :$

$AM + \frac{1}{2}MN :: \text{force of gravity} : \text{resistance in the middle of the fourth time}$. Therefore, $sMNt : smnt :: \text{force of gravity} : \text{resistance in the midst of the fourth time}$, and so of the rest.

37. And the like demonstration holds in ascending motion; for let the rectangle ABDG be divided into innumerable rectangles $Dk, Kl, Lm, Mn, \&c.$ which shall be as the decrements of the velocities produced in so many equal times; then will $AE, Ak, Al, Am, \&c.$ be as the whole velocities, and therefore as the resistances of the medium in the begin-

beginning of each of the equal times. Let $AC : \text{Fig. } AK$: : force of gravity : resistance in the beginning 37. of the second time; then to the force of gravity add the resistances, and $DEHC, KkHC, LlHC, MmHC, \&c.$ will be as the absolute forces, or \propto decrements of the velocities, $\propto Dk, K', Lm, Mn, \&c.$ and therefore \vdash ; whence $DGqK = KqrL = LrsM = MsN, \&c.$ will be \propto the equal times or forces. But $AC : MC : : Ms : AB.$ And by division $AC : AM : : Ms : ms : : \text{area } sMNt : \text{area } smnt : : (\text{and therefore as}) \text{ force of gravity : resistance in the fourth time}; \text{ and so of the other areas. Therefore since } DGqK, KqrL, LrsM, MsN, \text{ are } \propto \text{ the gravitating forces, the areas } GEkq, qklr, rlm, msnt, \propto \text{ resistances in each time, } \propto \text{ velocities } \propto \text{ spaces described. And by composition, in the times } DGrl, DGBA, \text{ the spaces described will be as } GEk, GEB. Q. E. D.$

[ib. cor. 3.] For the diff. spaces \propto velocities (that is by cor. 2.) $\propto CA, CK, CL (\vdash),$ or $\propto AK, KL, LM (\vdash)$ by lem. 1. (fig. 3.)

[ib. let that also be distinguish'd] This is true in this law of resistance; because the motion lost is \propto velocity \propto remaining velocity \propto spaces described.

[Pr. 4. which is the locus of the point $r.$ Q. E. D.] all the rest is plain by Prop. 2. and 3, and cor. Pr. 2.

[ib. cor. 1. that is, if the parallelogram] For $DA : CP : : DR : RX = \frac{DR \times CP}{DA}.$ But $DAB =$

$\underline{DC - AC \times AB} = (\text{by the nature of the hyperbola}) \underline{AB - AQ \times DC} = QB \times DC = (\text{by construction}) N \times CP.$ And $DA = \frac{N \times CP}{AB}.$ Where-

fore $RX = \frac{DR \times CP \times AB}{N \times CP} = \frac{DR \times AB}{N}$

[ib.

Fig. [ib. cor. 4. DraF is also given.] by this prop.
 [ib. cor. 6.] for then $2DP \propto \frac{\text{lat. rect.}}{\text{resistance}} \propto$

$$\frac{\text{lat.}}{\text{velocity}} \propto \frac{\text{velocity}^2}{\text{velocity}} \propto \text{velocity}.$$

[ib. cor. 7. the ratio $\frac{Ff}{DF}$] For when this ratio

is the same with the other, the curves described
 (and supposed to be described) in these two cases,
 will be similar.

S E C T II.

Prop. 5. had been better expressed thus, (tho' the translation agrees with the original.)

If a body is resisted in the duplicate ratio of its velocity, and moves by its innate force only, thro' a similar medium; and the spaces be taken equal: I say, that the times of their description are in a geometrical progression increasing; and that the velocities at the beginning of each of the times are in the same geometrical progression (decreasing or) inversely.

For it is plain, that the times may be taken in such a geometrical progression, as that the spaces cannot be equal, nor the velocities in the same inverse progression. But if the spaces are equal, the times and velocities will be in a geometrical progression, inverse to one another. The demonstration is the same however the proposition be expressed. For it being proved that the lines AB, Kk, Ll, &c. being squared, the squares are as the differences of the lines; and also that the squares of the velocities are as the differences of the velocities. Therefore if AB and Kk be taken as the velocities in the beginning of the times AK, KL; and an hyperbola be drawn thro' the points B, k, to the asymptote CD; (whose center let be C; all the other velocities will be as the lines Ll, Mm, &c. because

B. II. THE PRINCIPIA. 49

because the progression, of the lines as well as the Fig. velocities, continues the same all along. Whence the spaces (in these equal times) will be as the areas Ak , Kl , Lm , &c. and in the time AM ; the space will be as $AMmB$. Now conceive the area $AMmB$ to be divided into the equal areas Ak , Kl , Lm , &c. Then will CA , CK , CL , &c. be increasing; and the parts (by Lem. 1.) AK , KL , LM , &c. which are as the times, will be in the same progression, increasing. Also the velocities AB , Kk , Ll , &c. (which are reciprocally as CA , CK , CL , &c.) will be in the same progression inversely. Q. E. D.

[Pr. 7, and times conjunctly] very small particles of time.

[ib: they will always describe spaces,] for the parts of space described in those several particles of time, are as the respective velocities \times those several parts of time; that is, as the first velocity \times each part of time. And by composition, the whole spaces are as the first velocities \times whole times. Q. E. D.

[Prop. 7, cor. 3.] dele. Let those diameters applied to that power.

[Pr. 8.] in fig. 1, Pl. 2, k ought to be beyond l in respect of A , &c. because the resistance decreases. But the demonstration is the same any way. But this figure is not in the 1st or 2d Ed. of the original. In Prop. 8 and 9, AC represents the relative gravity, or its weight in the fluid.

[Pr. 9. cor. 7.] For the greatest velocity is given (by Pr. 8. cor. 2, 3.) And thence the time of acquiring that velocity in free space. And $ABNK$ ($ABnK$) being given there is given AK , and AP (Ap) and ATD (AtD). Then $ADC : ADT$ (ADt) :: time of acquiring that greatest velocity in free space : time sought.

[Pr. 10. equal among themselves] and very small.

D

[ib]

Fig. [ib. or $\frac{MI \times NI}{HI}$] as appears by similar Δ's, pro-

duced by letting fall a ⊥ from N on HI.

If the velocity in GH be greater than the velo-
city in HI, the decrement is $\frac{GH}{T} - \frac{HI}{t}$, and aris-

es from the resistance and gravity together (be-
cause gravity draws the body from the tangent in-
to the arch HI); and if gravity act not, that de-
crement would be greater (for gravity accelerates
the motion), by $\frac{2MI \times NI}{t \times HI}$; therefore the decre-

ment, by the resistance alone is $\frac{GH}{T} - \frac{HI}{t} +$
 $\frac{2MI \times NI}{t \times NI}$.

[ib. will be NI;] for Q being a given (ratio or)
quantity; Qo will always represent MN, whatever
the magnitude of o be.

And the ordinate D1 = CH - MI. Also the
value of MI in the ordinate EK is (because o be-
comes 2.o, and substituting $2 \times o$, for o,) $2Qo +$
 $4Ro^2 + 8Ro^3 + \&c.$ and therefore $EK = CH -$
 $2Qo - 4Ro^2 - 8Ro^3 - \&c.$ Also in the ordinate
BG, when o becomes - o, the value of MI (by
substituting - o for o) will be $-Qo + Ro^2 -$
 $So^3 + \&c.$ And $BG = CH - MI$ (as before,) =
 $P + Qo - Ro^2 - So^3 - \&c.$

[ib. or $\frac{R + \frac{3}{2}So}{R}$] for (because o is infinitely
small) R, $R + \frac{3}{2}So$, $R + 3So$, are both arithme-
tical and geometrical proportionals.

[ib. by substituting the values -] becomes
 $\frac{1 + \frac{3So}{2R} \times o \sqrt{1+QQ}}{\sqrt{1+QQ}} - \frac{QRoo}{\sqrt{1+QQ}} - o \sqrt{1+QQ}$

B. II. THE PRINCIPIA.

51

$$-\frac{QRoo}{\sqrt{1+QQ}} + \frac{2Qo+2Ro^2 \times Ro^2 + So^2}{o\sqrt{1+QQ} + \frac{QRoo}{\sqrt{1+QQ}}} = (\text{neglecting the superfluous powers of } o, \text{ or the quantities infinitely less than the other,})$$

Fig.

$\frac{QRoo}{\sqrt{1+QQ}} - o\sqrt{1+QQ} -$

$$\frac{QRoo}{\sqrt{1+QQ}} + \frac{3So^2}{2R} \sqrt{1+QQ} - o\sqrt{1+QQ} -$$

$$\frac{QRo^2}{\sqrt{1+QQ}} + \frac{2QRoo}{\sqrt{1+QQ}} = \frac{3So^2}{2R} \sqrt{1+QQ}.$$

[ib. latus rectum $\frac{HN^2}{NI}$] or $\frac{HI^2}{NI} = (\text{neglecting the superfluous powers of } o)$

$$\frac{o \times 1 + QQ}{Roo} = \frac{1+QQ}{R}$$

[ib. p. 33, that a body by ascending from P] or any other point of the quadrant PF, in the direction of the arch of that quadrant.

[ib. Ex. 3. the second term $\frac{m}{n}o - \frac{bb}{aa}o$] or rather $\frac{bb}{aa}o - \frac{m}{n}o = Qo$; but the square is the same either way.

$$[\text{ib. Ex. 4. for } Qo] \frac{nbb}{A^{n+1}}o - \frac{d}{e}o = Qo.$$

[ib. Sch. And therefore if a curve] for the quantity $\frac{s}{R^{4-e}} \div \frac{HT^{n-1}}{AC}$ is as the density, and if that be given the density is given.

[ib. than these hyperbola's here described.] For let the 2 mediums be of the same density in the vertex of the curves; then if the bodies be projected from A; the uniform medium being more dense, and the other less dense at A, the body moving in the uniform medium will be more impeded,

D 2

and

Fig. and consequently will descend the more, than that describing the hyperbola, which is less resisted, and which therefore moves nearer a straight line. Therefore the body moving in the uniform medium is more distant from the asymptotes in the vertex of the figure than the other body. But if the densities of the mediums were supposed equal in A ; then (by the same way of reasoning), the body in the uniform medium would be nearer the asymptotes in the vertex. And generally speaking, these curves cannot therefore differ very far.

[ib. will touch the hyperbola in G,] for suppose the tangent TG to intersect MX. Then (by fluxions) the distance between that point of intersection and V, is $= \frac{VX}{n}$. Then by sim. Δ's, $\frac{VX}{n} : VG$
 $\therefore VX : TX - VG = nVG$. But $VY = nVG$. Therefore $TX = GY$.

[ib. and the velocity] for the velocity (when the gravity is given) is as the ordinate of the parabola, that is (the abscissa being given, by the gravity) as the $\sqrt{\text{latus rectum}}$.

[ib. Rule 1.] For the velocities are as the $\sqrt{\text{'s}}$ of the latus rectum of two parabola's, that is, as $\sqrt{\frac{2XY^2}{nn+n\times VG}}$, that is, as $\sqrt{\frac{2AH^2}{AI}}$ to $\sqrt{\frac{2Ab^2}{Ai}}$ which (by supposition) are in a ratio of equality, and therefore $\frac{AH}{AI} = \frac{Ab}{Ai}$. Also since the densities are equal ; $\frac{n+2}{3XY}$ will be given, that is $\frac{n+2}{3AH}$

$\equiv \frac{n+2}{3Ab}$. Whence $AH = Ab$. Whence AH , AI , remain the same ; and HX , which is composed of AI and the subtangent to the axis XH , is the same also.

[Rule

B. II. THE PRINCIPAL.

53

[Rule 2.] For velocity $\propto \sqrt{\frac{AH^2}{AI}}$ \propto (because Fig. AH is given) $\sqrt{\frac{I}{AI}}$.

[Rule 3. and therefore AH] for let b be the ratio, then $\frac{Ab}{Ai} = b \frac{AH}{AI}$. And $\frac{AH^2}{AI} = \frac{Ab^2}{Ai} = b \frac{AH}{AI} \times Ab$. Whence $Ab = \frac{AH}{b}$. Also $\frac{Ab}{Ai} = \frac{AH}{bAi} = b \times \frac{AH}{AI}$. Whence $Ai = \frac{AI}{bb}$.

[Rule 4. a little greater] since AH decreases faster than GT, therefore the sum of all or every AH + GT is lesser than the first AH + GT \times number of them. And therefore the sum of all the densities (being reciprocal thereto) is greater than the sum of as many densities in A and G. But the mean density, is = sum of all the densities \div by the number of them, and therefore is something greater than the $\frac{1}{2}$ sum of the densities in A and G. But half sum densities : density in A :: $\frac{AH + GT}{2}$; GT. Therefore mean density : density in A :: is in oratio a little greater than $\frac{1}{2} \times \frac{AH + GT}{2}$: GT.

[ib. Rule 5.] for HX = AI + subtangent = AI + nAI. XN being the axis.

[ib. Rule 6. by how much] for the variation of curvature in one of these hyperbola's, where n is great, is least in the part AG, and so it is in the curve of projection, where the velocity is greatest; so they agree in that part. But they differ more in the part GK, where the projectile approaches to a uniform motion.

[ib. Rule 7.] AH to 2AI, ought to be 3AH to 4AI. (But the original is AH to 2AI.)

D 3

{ib.

A C O M M E N T o n

Fig. [ib. Rule 8. whose conjugate] hyperbola shall pass thro' the point C.

[ib. what has been said] for by supposing the index n to be negative, VG (which before was as VX^{-n}) will then be as VX^n , and the curve AG will be a parabola, and what was demonstrated generally of the hyperbola, will hold true (for any index, and therefore) for the parabola; and all things will follow as before substituting $-n$ for n .

38. But the computation may be made for the parabola, as for the other; thus, let $VG = bbXV^n = XC^n$. Then the subtangent $ZT = nVG$. And let $XC = a$, $ng = e$, $C = 0$. Then $VG = \frac{a+e^n}{a+0^n} = a^n + na^{n-1} 0 + n \times \frac{n-1}{2} a^{n-2} 00 + n \times \frac{n-1}{2} \times$

$$\frac{n-2}{3} a^{n-3} 0^3 = P - Q_0 - R_{00} - S_0^3. \text{ Whence}$$

$$Q = -na^{n-1}, \quad R = -n \times \frac{n-1}{2} a^{n-2}, \quad S = -n$$

$$\times \frac{n-1}{2} \times \frac{n-2}{3} \times a^{n-3}. \text{ Therefore the density}$$

$$\text{in } G \text{ is as } \left(\frac{S}{R\sqrt{1+QQ}} = \right) \frac{n-2}{3a\sqrt{1+n^2a^{2n-2}}}$$

that is, as $\frac{1}{\sqrt{a^2+nna^{2n}}}$, or as $\frac{1}{\sqrt{a^2+nne}}$, that is

reciprocally as $\sqrt{XC^2 + TZ^4}$. And the same may be found by fluxions, putting Q, R, S, for the 1st, 2d, 3d, fluxion of VG or XC^n .

S E C T. III.

[Pr. 12, and inversly as the velocity; for the time, of describing a given space, is reciprocally as the velocity.

[Pr. 13. case 1, 2, 3, the decrement or increment PQ] in a given time.

[ib.]

[ib. cor. and triangle do.] therefore *ex equo*, &c. Fig. [ib. Sch.] For draw the indefinite line BAP, 39. and make BD perp. and equal to BA; and draw DF, AF parallel to BA, BD. Let AP be the velocity, $AP^2 + 2BAP$ the resistance, AB^2 the force of gravity. Draw DTP, cutting FA in T, and the time of the whole ascent will be as the triangle DTA.

For draw DVQ cutting off PQ the moment of velocity, and DTV the moment of the triangle; then the decrement of the velocity PQ will be as the resistance and gravity, $AP^2 + 2BAP + AB^2$; that is as BP^2 . But the area DTV is to the area DPQ, as DT^2 to DP^2 , or as DF^2 to BP^2 ; therefore since the area DPQ is as BP^2 , the area DTV will be as the given quantity DF^2 . Therefore the area ADT decreases uniformly as the time, by the subduction of given particles DTV; and therefore is proportional to the whole time of ascent.

[Pr. 14. From the area DET] This should be; from the moment KLOM, subduct DTV or $m \times BD$ the moment of DET; and is only a false translation.

[ib. cor. or as V^2 ;] for the space is $\propto \square$ of the time, that is, \propto the square of DET or of BD \times M \propto (because $M = \frac{DE \times V}{DA}$) square of $\frac{BD \times DE \times V}{DA}$, that is (because BD, DE, DA are given) as V^2 , or as $\frac{BD \times V^2}{AB}$.

[ib. Sch. instead of the uniform resistance made to an ascending body;] this differs from gravity, only in this; that it cannot generate any motion; but it acts after the same manner in all moving bodies in destroying their motion, as gravity does in destroying the motion of ascending bodies. Gravity acts uniformly in a given direction. The force

Fig. arising from tenacity acts uniformly, but always in a direction contrary to the motion of the body. And therefore when the body is at rest, it can induce no change in it. Now in the horizontal motion of a body in a fluid, which is resisted in part uniformly, one may substitute the force of gravity for that uniform resistance, as in Pr. 8, 9; 13, 14. And in the ascent or descent of the body in a fluid, instead of the force of gravity, one may substitute the sum or difference of that uniform force and the force of gravity, as in these propositions.

S E C T. IV.

In Prop. 15.

[And the decrements of these arcs arising from the resistance, will be as the squares of the times in which they are generated ;] for since the arcs are infinitely small, the resistance affects the same as an uniform centripetal force would do.

40. [ib. The decrement of the arch PQ will be] for let q, t be the points it would arrive at, in the same times, in free space, then $Pq = qt$,

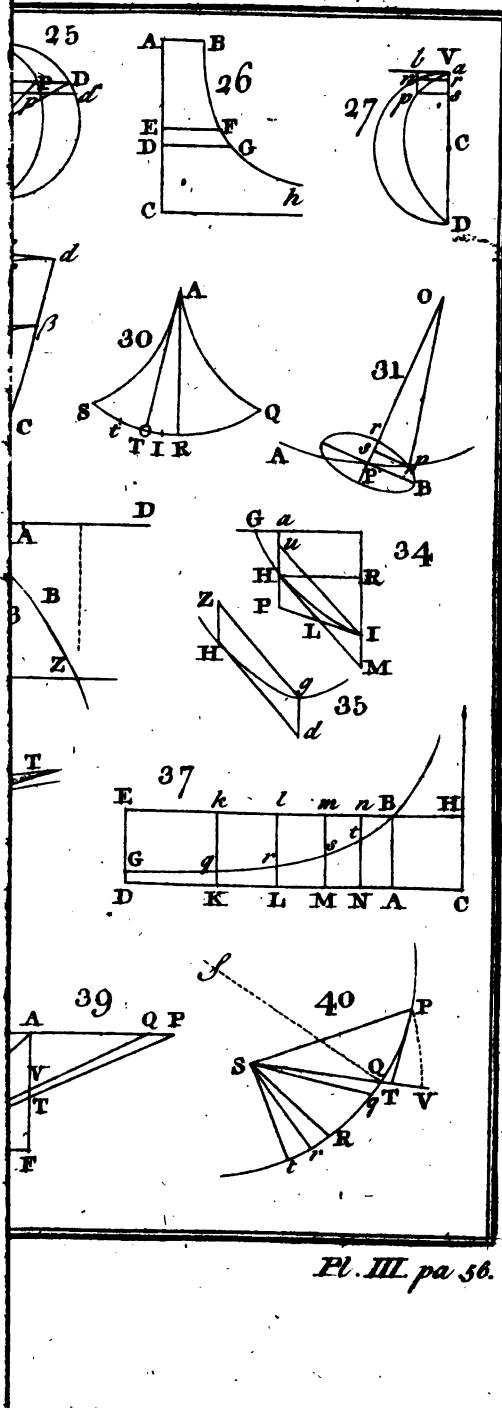
$$\text{and } PQ = Qr, \text{ therefore}$$

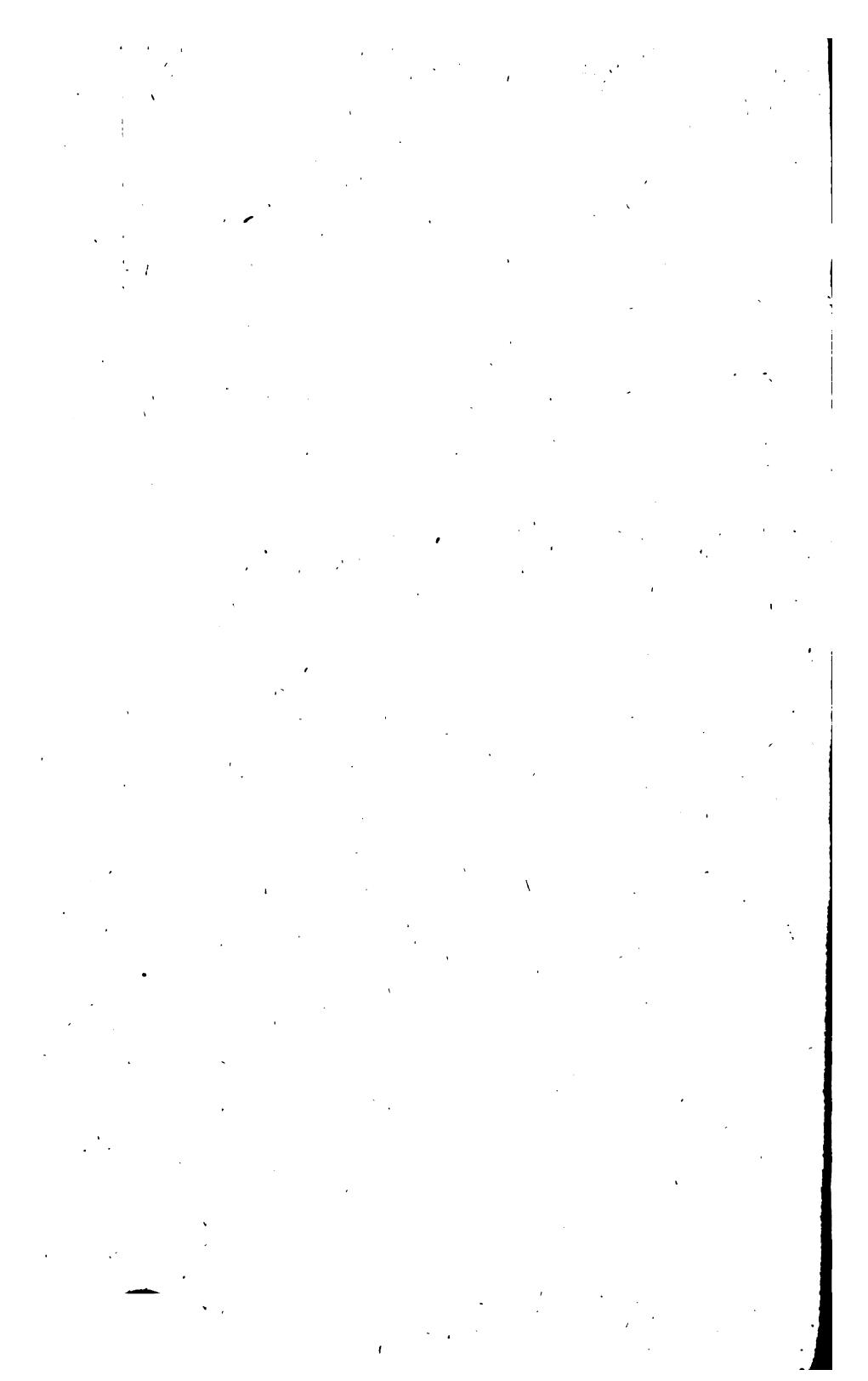
$Qq = qt - Qr$; and adding Qq , $2Qq = Qt - Qr = tr$; and $Qq = \frac{1}{2}tr$: but $Qq = \frac{1}{4}Rt$; therefore $tr = \frac{1}{2}Rt = Rr$, and $Qq = \frac{1}{2}Rr$.

The same demonstration of this Prop. holds good, as well in ascending as in descending motion.

41. [Cor. 1. The velocity, &c.] for supposing PQ an arch of a circle whose center is f ; then if PQ or PT be as the velocity in both curves, TQ will be as the centripetal force in both, but in both curves $TQ = \frac{PQ^2}{2Pf}$; therefore $Pf = \text{rad. of the circle described with the same velocity.}$

[Cor.





B. II. THE PRINCIPIA.

57

[Cor. 4. the body will descend, &c.] for by cor. Fig. 1, the velocity in P in a resisting medium is \equiv velocity in a circle at distance SP in free space; but velocity at distance SP in free space : to velocity at dist. $\frac{1}{2}SP$ in free space (by cor. 6. Prop. 4. B. I.) or Prop. 34. :: as $\sqrt{\frac{1}{SP}}$: to $\sqrt{\frac{1}{\frac{1}{2}SP}}$:: that is, $\sqrt{\frac{1}{2}}$; therefore, &c.

[Cor. 7, or as $\frac{1}{2}AS$ to AB] (for this see my Geometrical Proportion, prop. 24.)

In Prop. 16.

[And the resistance, &c.] instead of this proportion in prop. 15, viz. PQ : QR :: SQ : $\sqrt{SP \times SQ}$, substitute this PQ : QR :: $(SQ^{\frac{2}{2}} : SP^{\frac{2}{2}} ::) SQ : SQ + \frac{n}{2} SP - \frac{n}{2} SQ$, (by prop. 24, of my Geomet. Proportion) that is, as the describing velocities, &c.

[Cor. 1.] For the resistance : centripetal force :: $(\frac{1}{2}Rr =) \frac{1 - \frac{1}{2}n \times VQ \times PQ}{2SQ} : (TQ =) \frac{\frac{1}{2}PQ^2}{Sp} :: \frac{1 - \frac{1}{2}n}{VQ} VQ : PQ :: 1 - \frac{1}{2}n OS : OP$.

S E C T. V.

In Prop. 19.

[Case 3, different spherical parts have equal pressures,] that is, different spherical parts of equal magnitude.

In Prop. 20.

[And by a like reasoning, &c.] Supposing the thickness of the orbs to be reciprocally as the force of gravity or the density of the fluid, or in the complicate ratio of them; and it will appear (as before)

Fig. before) that the bottom is pressed with a cylinder, of the same fluid; whose base is = the bottom, and height that of the fluid, by the same reasoning as before, and prop. 19.

In Schol. Pr. 22.

[By a like reasoning, &c.] Let the force of gravity be reciprocally as the n th power of the distance. The specific gravities at A, B, C, &c. will

$$\propto \frac{AH}{SA^n}, \frac{BI}{SB^n}, \frac{CK}{SC^n}, \text{ &c. and the densities } \propto$$

sums of the pressures $\propto \frac{AH \times AB}{SA^n}, \frac{BI \times BC}{SB^n},$

$$\frac{CK \times CD}{SC^n}, \text{ &c. } \propto \frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}, \frac{CK}{SC^{n-1}}, \text{ &c.}$$

And $tu, uw, \text{ &c. } \propto$ differences of the densities \propto

$$\frac{AH}{SA^{n-1}}, \frac{BI}{SB^{n-1}}, \text{ &c. And } tu \times tb, uw \times ui \propto$$

$$\frac{AH \times tb}{SA^{n-1}}, \frac{BI \times ui}{SB^{n-1}} \propto \frac{I}{SA^{n-1}}, \frac{I}{SB^{n-1}}, \propto \frac{I}{SA^{n-1}} -$$

$$\frac{I}{SB^{n-1}} - \frac{I}{SC^{n-1}}, \text{ &c. and therefore universally }$$

$$\frac{I}{SA^{n-1}} - \frac{I}{SE^{n-1}} \propto \text{ hyperbolic area } tbmy.$$

$$\text{Therefore if } \frac{SA^n}{SA^{n-1}}, \frac{SA^n}{SB^{n-1}}, \frac{SA^n}{SC^{n-1}}, \text{ or } \frac{I}{SA^{n-1}},$$

$$\frac{I}{SB^{n-1}}, \frac{I}{SC^{n-1}}, \text{ be } \ddots \text{; then } \frac{I}{SA^{n-1}} - \frac{I}{SB^{n-1}} =$$

$$\frac{I}{SB^{n-1}} - \frac{I}{SC^{n-1}}, \text{ and the areas proportional thereto}$$

will be equal, viz. $tbiu = uikw$, and therefore S_t, S_u, S_w , that is, AH, BI, CK \ddots . Q. E. D. then putting n successively $= 3, 4, 0 - 1, \text{ &c.}$ the truth of the said Scholium will appear. And so on, ad infinitum.

[Other]

B. II. THE P R I N C I P I A. 59

[Other laws of condensation, &c.] Suppose the Fig. compressing force \propto rth power of the density. Then the specific gravities in A, B, &c. \propto $\frac{AH}{SA^2}$, $\frac{BI}{SB^2}$, &c. And the pressures in A, B, &c.

$$\propto \frac{AH}{SA} + \frac{BI}{SB} + \frac{CK}{SC} + \frac{DL}{SD} \text{ &c.}, \frac{BI}{SB} +$$

$$\frac{CK}{SC} + \frac{DL}{SD}, \text{ &c.}$$
 Now if these sums were $\underline{\underline{=}}$, their differences $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. would be in the same $\underline{\underline{=}}$, and then the pressures in A, B, &c. $\propto \frac{AH}{SA}$, $\frac{BI}{SB}$, &c. \propto densities' $\propto AH'$, BI' , &c. And therefore AH'^{-1} , BI'^{-1} $\propto \frac{1}{SA}$, $\frac{1}{SB}$. And AH , $BI \propto \frac{1}{SA^{-1}}$, $\frac{1}{SB^{-1}}$. Then putting $r = \frac{4}{3}$, $\frac{5}{3}$, 2, &c. all things will appear as in this Schol. But now the aforeaid sums are not in geometrical progression, because it is plain their differences are not, viz. $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. for if $\frac{AH}{SA}$, $\frac{BI}{SB}$, &c. $\underline{\underline{=}}$, and seeing SA , SB &c. are $\underline{\underline{=}}$, therefore (by Prop. 18. cor. 2. of my Geom. Proportion) AH , BI , &c. would be $\underline{\underline{=}}$ which is false (except in the case of Prop. 21.) therefore neither these differences, nor sums are $\underline{\underline{=}}$. And consequently the conclusions of this Schol. in this respect are not true. And therefore it must be presumed, that the Author meant them only to be nearly, and not perfectly true.

In

A C O M M E N T o n

In Schol. Pr. 23.

[All these things are, &c.] It seems to me that the prop. holds true, tho' the centrifugal force be extended to all distances, in any given law, as well as when it ends at the next particle in the same given law. For supposing the particles of the fluid to be placed in parallel planes, and that these planes act on each other; the force of compression on any one plane will be made up of the several forces, of all the other equidistant parallel planes. Therefore since the distances of all these planes before compression are respectively proportional to their distances after compression from this given plane; their actions on the given plane before and after compression will also be proportional, (seeing their actions or forces are as some power of the distances); and their sums will be also proportional to those of any two corresponding planes, viz. as the nearest to the nearest. In these sort of particles there will be required a greater force to produce an equal condensation of an equal quantity of the fluid. But the density will be proportional to the compression, as before.

S E C T VI.

In Prop. 24.

Cor. 5. — for let m , M = quantities of matter, w , W = weights, t , T = times, for the common pendulum l . Also L = new pendulum. T = new time, for M , and W . Then m : $M :: wtt : WTT$ (by the Prop.) : : $\frac{wtt}{t} : \left(\frac{wtt}{L}\right) =$

$\frac{WTT}{L}$. And if m , M ; t , T , are equal; $w \propto l$, which is cor. 4.

In

In Prop. 27.

Fig.

[If the resistance in the arc B, &c.] If the resistance in the arc A were to the resistance in the arc B as AA to AB, the times would be equal, and therefore resistance AA in the arc A causes the excess of time above that in a non-resisting medium; and resistance AB in the arc B causes the excess of time in B above that in a non-resisting medium, equal to the former excess, (because with these resistances) they are described in equal time. But now if B be described with a resistance BB; the excess of time will be caus'd by that resistance.

[But those excesses, &c.] Now the excesses of time caused by describing B with the resistances AB, BB respectively, are as the spaces left undescribed after the given time; that is, as the resistances AB, BB nearly, or as A to B, viz. as the arches A, B. And since the excess of time by describing A with resistance AA = excess of time by describing B with resistance AB. Therefore it follows, that excess of time in A : to excess of time in B : as A : to B nearly.

In Prop. 29.

[Area PIEQ may be to the hyperbolic area PITs :: as BC to Ca; and that the area IEF may be to the area ILT as OQ to OS.]

That this is possible (assuming P at pleasure) may be thus shewn. Let the areas PIEQ, PITs be taken (as BC to Ca) indefinitely small, then PIEQ must be greater or at least equal to PITs; suppose them equal, then will PIFQ > PILS, and IF > IL (and more so, if $\text{PIEQ} > \text{PITS}$), but when S, P, Q coincide OS to OQ is a ratio of equality, therefore in this case IFE to ILT (or IF to IL) > OQ to OS (or however not less), increase the areas PIEQ and PITs in the same ratio, and

the

Fig. the ratio of OQ to OS will also increase and converge at length to the ratio of IEF to ILT; which shews the possibility of what was required; (but as to an actual solution I shall refer it to an algebraic process.) This being done as required; then, the points O, S, P, Q are all fixt, and the point R only variable.

[And the increment, &c.] for, the fluxion of $\frac{IEF}{OQ}$ OR — IGH to the fluxion of — PIGR :: as $\frac{IEF}{OQ}$ — HG to — RG :: or as HG — $\frac{IEF}{OQ}$ to RG.

Or rather thus, the increment of $\frac{IEF}{OQ}$ OR—IGH, is as $\frac{IET}{OQ}$ — HG × — Rr = HG × Rr — $\frac{IEF}{OQ}$ × Rr.

And the increment of PIGR is as RG × — Rr; and therefore the decrement (because the decrement is a negative increment) of PIGR, is as RG × Rr.

In cor. to Pr. 30.

[Now if the resistance DK be in the duplicate, &c.] for the velocities being as the ordinates of a circle or ellipsis described on aB; the resistances (being as the squares of these velocities) are as the squares of these ordinates, that is, as the rectangles of the abscissa's. Therefore the resistances DK, OV are respectively as aDB, (aOB =) OB². But this is the property of the parabola aVB described to the axis VO, by cor. 2. to p. 4. of the parabola.

Gen. Sch. Pr. 31, pa. 96.

[But in lesser oscillations somewhat greater, &c.] that is, the difference in the lesser arc to the difference in the greater arc, is in a greater ratio than the squares of these lesser and greater arcs.

[pa.

[pa. 97.] for $\frac{1}{2}$ the cords of these arcs, read the Fig. cords of those half arcs.

$$[\text{ib. p. 98. As } \frac{7}{11} AV + \frac{7}{10} BV^{\frac{3}{2}} + \frac{3}{4} CV^2.]$$

These three members are taken to denote three laws of resistance, viz. the first in the simple ratio of the velocity, the third in the duplicate ratio thereof, and the second partly in the simple partly in the duplicate ratio (or in a mean between them); and $\frac{7}{10}$ the coefficient thereof is a middle one between $\frac{7}{11}$ and $\frac{3}{4}$.

[As 0,041748 to 121] it should be as 0,041778 to 121.

[pa. 99. It is manifest that the force, &c.] for the forces of resistance and gravity are as their effects, to wit, as the velocity lost (by resistance) and velocity gained (by gravity) in the same time, as 1 to $376\frac{1}{5}$.

[I also counted—I leave the calculation, &c.] the calculation after the preceding manner will be as follows.

Mean oscillations	—	$3\frac{1}{2}$	7	14	28	56	112
diff. between 1st def. and last as.		$\frac{1}{2}$	1	2	4	8	16
diff. of arcs described in the de- scent and subsequent ascent.	3	$\frac{1}{748}$	$\frac{1}{272}$	$\frac{4}{325}$	$\frac{12}{250}$	$\frac{24}{125}$	$\frac{48}{68}$

Also these in the greater arches are nearly in the duplicate ratio of the velocities, but in the lesser arches somewhat greater, (these arcs are in the last series of the table).

Let V = velocity in 2d, 4th, and 6th case = 1, 4, 16 respectively. And the difference of the arcs will be, in the

$$2d \text{ Case} = \frac{1}{V^{\frac{1}{2}}} = A + B + C$$

$$4th \text{ Case} = \frac{1}{V^{\frac{2}{3}}} = \frac{1}{V^{\frac{6}{5}}} = 4A + 8B + 16C$$

$$6th \text{ Case} = \frac{1}{V^{\frac{3}{2}}} = \frac{1}{V^{\frac{12}{7}}} = 16A + 64B + 256C.$$

whence $A = .0005098$; $B = .0005882$; $C = .0025784$. Therefore the difference of the arcs

A C O M M E N T O N

Fig. arcs is as ,0005098V + ,0005882V $\frac{1}{2}$ + ,0025784V $\frac{3}{2}$. And consequently the resistance of the globe in the middle of the arc, will be to its weight (by cor. pr. 30.) as ,00032442V + ,00041174V $\frac{1}{2}$ + ,0019338V $\frac{3}{2}$, to 121. Therefore the resistance will be to the weight, in case 2d, 4th, and 6th, as ,00267 to 121; ,0355324 to 121; ,5265949 to 121, respectively.

Note, you may take any other numbers (that are in the same proportion with these above) for V; for example I took $\frac{1}{4}$, 1, and 4 = V, in the 2d, 4th and 6th cases, and A will be = ,002039; B = ,004706; C = ,041255; and the resistance to the weight (at the last) comes out the same as before.

In the 6th case the point marked in the thread described an arc of $112 - \frac{4}{8}^{\frac{1}{2}}$ inches = $111\frac{5}{7}$; therefore the center of the globe describes an arc = $115\frac{167}{87}$ = $115\frac{8}{5}$ nearly; and its velocity is (nearly) the same as in descending an arc $57\frac{1}{4}$ of a cycloid (whose semi-arc is 126,) or = velocity acquired by falling perpendicularly thro' the versed sine (or abscissa) of that arc $57\frac{1}{4}$, but this versed sine is = 13,324. Therefore the velocity of the pendulum is = velocity acquired by falling perpendicularly 13,324 inches. And with this velocity, the globe meets with a resistance which is to its weight as ,52659 to 121, or (taking that part only of the resistance which is as the square of the velocity,) as ,49505 to 121.

Also if a globe of water of equal magnitude moves with the same velocity, its resistance will be to its weight as ,49505 to 213,4; or as 1 to $43\frac{1}{4}$. Whence, in the time in which the globe uniformly describes 26,648 inches, the weight of the globe of water will generate all that velocity in the falling globe; therefore the velocity destroyed by

by resistance, will be to that acquired by gravity Fig. (in the same time), as 1 to $431\frac{1}{3}$, or velocity lost

$= \frac{1}{431\frac{1}{3}}$ of the whole velocity. And therefore in the time it would uniformly describe its semidi-ameter, with the same velocity, it would lose the $\frac{1}{334\frac{1}{3}}$ part of its motion.

334¹.7

[pa. 101, 2, after five oscillations] this certainly should be after 10 oscillations, as appears by the process of the calculation.

[pa. 102. So that the difference 0,4475, &c.] for the motion lost is \propto the resistance, and that \propto the square of the velocity and square of the time nearly, which in this case is a constant product.

[pa. 103, l. 6.] Here is a small error, viz., 61675 instead of 61705; whence 45,453 would have been 45,433; and the resistances as 7,002 to 1; but this is not material.

[pa. 104. because the resistance, &c.] for the resistance \propto square of the time \times square of the velocity, which product is invariable, because equal arches are always described.

S E C T. VII.

[Pr. 32. ratio of density] he means any given ratio of density, which are proportional to the particles.

[Pr. 32. Proportional times] any times, in that constant ratio, that is, let them move among one another, in similar directions, with velocities which are as the particles.

[Pr. 32. cor. 2. spaces proportional to their diameters] in these proportional times.

[Pr. 33. cor. 2. for if the forces, &c.] for the resistance arises from the centrifugal forces, and from the collisions, of the particles. The resistances of the first sort are as the motive forces,

E that

Fig. that is, as the accelerative forces and quantity of matter, that is, (by supposition) as the squares of the velocities and quantities of matter, that is, (because the quantities of matter are given, the fluid being the same) as the squares of the velocities.

The resistances of the second sort are as the number of reflexions, and their forces; that is, (as is proved in the Prop.) as the squares of the velocities, squares of the diameters, and densities of the parts, that is (because the diameters and densities are given) as the squares of the velocities, accurately.

[As also the bodies E and G;] let E and G be vastly swifter than D and F.

[Pr. 34. but the former of, &c.] for $bH \times CB = (BE^2 =)ObN$, which is the property of the parabola.

[Pr. 34. Schol less resisted than the former solid;] for, if FG and IH be produced till they intersect, they will form a right angle; and therefore the frustum FGBHI will be less resisted than if the lines should intersect in an obtuse angle, (as is shewn in the cone;) which they would do if the lines fell within FG and IH; and for the same reason the resistance would be greater in the curve itself, because the lineoleæ which constitute it, will, when produced, intersect one another at obtuse angles. What is mentioned besides in this Schol, is demonstrated in the appendix.

[Pr. 35. it follows, that the cylinder, &c.] for mot. cylinder : mot. medium :: cylinder's magnitude \times density \times velocity : (medium's mag. \times denſ. \times vel. $=$) cylinder's $\frac{\text{magn.}}{2} \times 2 \text{ vel.} \times \text{medium's}$

density :: cylinder's density : medium's density,

[ib. to the force by which] to the motive force by which its whole motion, &c.

[Pr. 35. cor. 2.] for, the resistance of the globe and the force (that will quite take away its motion in the time it moves two or four thirds of its diameter,) \propto

ameter,) are in a given ratio; therefore increase Fig. the velocity of the globe in any ratio, and the force (that will utterly destroy all its motion in a given space) will be increased in a ratio of the velocity directly and the time inversely, that is, as the square of the velocity, therefore the resistance of the globe is increased in the duplicate ratio of the velocity.

[ib. cor. 3.] for the same reason, increase the diameter of the globe in any ratio, and the motive force (that will utterly destroy its motion) will be increased in the triplicate ratio of the diameter directly, and the time inversely, that is, in the triplicate ratio of the diameter directly, and the simple ratio of the diameter inversely, that is, as the square of the diameter; and therefore the resistance of the globe will be increased in the duplicate ratio of the diameter.

[Cor. 6. and its resistance, &c.] Let BC be the resistance at first, then resistance at the first : resistance at the end :: $BC^2 : EF^2$; but $AE : AB :: BC : EF :: EF : BH$; therefore $BC^2 : EF^2 :: BC : BH$. ∴ BH is the resistance at the end.

[Cor. 7. As the logarithm, &c.] For (by Schol. 42. Prop. 86, hyperbola,) it appears, that if $AH = 1$. $AB = T$. $BK = t$: then it will be $CBKF : EHKF :: (\log. T + t - \log. T) : \log. \frac{T+t}{T} : \log. \overline{T+t}$.

And $EHKF : AHED :: (\log. \overline{T+t} : ,43429448 ::)$
 $\log. \overline{T+t} \times 2,30258509 : 1$. Also $AHED : CBKG :: HE : CB \times t :: T : t$. Therefore *ex equo*,
 $CBKF : CBKG :: \log. \frac{\overline{T+t}}{T} \times 2,30258 \times T : t ::$

$$\log. \frac{\overline{T+t}}{T} \times 2,30258509 : \frac{t}{T}$$

[Pr. 36. cor. 2.] Forces are equal when their effects are equal in a given time; but the effect of the motion of the effluent water (in the time a body

Fig. dy descends thro' GI) = a cylinder, whose length is $2GI$, and base the hole EF; also the effect of the gravity or weight of a column whose height is GI , and base the hole EF = (by reason of an accelerated motion, and in the same time) a cylinder whose length is GI , and base the hole EF; and therefore the effect of the pressure of twice that column (in the same time) = a cylinder, whose length is $2GI$, and base the hole EF; = the first cylinder, ergo, &c.

[Cor. 3. or $IH + IO$ to $2IH$.] for $ib : bo :: io : og$; or $bo : og :: ib : io$. And $bo + og : 2bo :: ib + io : 2ib :: io + ig : 2io :: \odot EF + \odot AB : 2 \odot EF$.

[ib. cor. 10.] This cor. ought to be more exactly computed and demonstrated, because the following propositions depend thereon.

[Pr. 37. pa. 137. And is therefore nearly equal, &c.] This resistance = weight of a cylinder (whose base is that little circle and altitude $\frac{1}{2}IG$, from which altitude the cylinder must fall, &c.) = force by which its motion may be generated, &c.

[Pr. 37. cor. 1.] Let F = force (mentioned in the prop.) M = medium's density. C = cylinder's density. V = velocity. D = diameter. T = time (of describing four times its length). R = Resistance. Then $R = F \times \frac{M}{C}$ (by the prop.) $\propto C \times$

$$\frac{V}{T} \times D^2 \times \frac{M}{C} \propto V^2 D^2 M.$$

[Cor. 2.] Let EF be given, and density of the medium be also given. Then because this resistance = weight of a cylinder (whose base is PQ, and height $\frac{1}{2}IG$) $\times \frac{EF^2}{EF^2 - \frac{1}{4}PQ^2}$; therefore weight of this cylinder = resistance $\times \frac{EF^2 - \frac{1}{4}PQ^2}{EF^2}$. But the force requisite to take away the motion of this cylinder,

cylinder, whilst it moves four times its diameter Fig.
 ∞ weight of this cylinder \times velocity \times

$$\frac{\text{time of describing 4 times its length}}{\times \frac{EF^2 - \frac{1}{2}PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2} \times \frac{EF^2 - PQ^2}{EF^2}} \propto \text{resistance}$$

And the density of the cylinder being increased, the force is increased in the same ratio.

[Cor. 3.] for since ref. : force :: $EF^6 \times \text{density}$
 medium : $\frac{EF^2 - \frac{1}{2}PQ^2}{EF^2} \times EF^2 - PQ^2 \times \text{density}$
 cylinder :: med. density : $\frac{EF^2 - \frac{1}{2}PQ^2 \times EF^2 - PQ^2}{EF^6}$
 \times cyl. density ; increase the distance it moves, in any ratio ; and the force (to destroy the motion) will be decreased in that ratio, as suppose the ratio of $\frac{EF^2 - \frac{1}{2}PQ^2 \times EF^2 - PQ^2}{EF^6}$. Then ref. : force ::
 med. density : cylin. density.

[Sch. pr. 37. AE and BE be 2 parabolic arcs] 43.
 since the resistance is the same, whether the fluid or the cylinder move, therefore the case of the resistance of the circle in fig. 4. is the same case with the resistance of the circle AB in fig. 6, considering the velocities HG and EG, wherewith they are acted on ; but (by cor. 9, pr. 36.) the resistance on PQ = $\frac{1}{2}$ cylinder on PQ and height GH (thro' which a falling body acquires the velocity, which the fluid has on PQ), but $\frac{1}{2}$ this cylinder is = a parabolic spindle on PQ and height GH, nearly. Therefore the resistance on AB = $\frac{1}{2}$ cylinder on AB and height GE (= GH,) thro' which a falling body acquires the velocity with which the cylinder moves,) which is equal to a parabolic spindle on AB and height GE = AEB by construction, (since lat. rect : (HG =) GE : : GE : AG.) This is the very construction of fig. 2.

Fig. [ib. must be to this force as 2 to 3, at least,] for the weight upon the little circle (in cor. 7. p. 36.) is there shewn to be equal to a cylinder of $\frac{1}{3}$ the height GH at least. But the weight of a cylinder on that circle and $\frac{1}{2}$ that height is the force whereby the cylinder's motion may be generated in the time it moves 4 times its length or $2GH$; therefore the resistance in that cor. is to the weight here demonstrated (*ceteris paribus*) that is, to the force :: as $\frac{1}{3}$ to $\frac{1}{2}$:: that is as 2 to 3, at least; and increasing the density, the force is increased in the same ratio.

[ib. let CF and DF.] It is necessary to allow as much time for the meeting of the particles of water (at the axis of the solid from the side thereof), after they are past the solid, as for their separation before they come to it. Now suppose this to be the case of fig. 4. In the time that a particle passes from H to P, the same particle with the velocity in P, will in the same time describe $2HG$, and therefore must pass twice as far to arrive to the axis again. Whence in fig. 6, the particle F must be twice as far from CD, as E from AB; but the height of FCD = 2 height of AEB, by construction. For let z , y = 2 ordinates to $x = \frac{1}{4}AB$; and $zz = 4rx = 4yy$; and $z = 2y$.

[Cor. 1. Pr. 38.] the demonstration is the same with that of cor. 1. pr. 37.

[Cor. 2. Pr. 38.] Let density of globe : dens. of fluid :: d : 1. Then $d \times \frac{4}{3}$ diameter of the globe = space fallen, and $d \times \frac{8}{3}$ diam. globe = space it describes afterwards, in the time of its fall. Then, because forces (which generate a given motion), are reciprocally as the (times, that is reciprocally as the) spaces wh^{ch} any body describes with the same uniform velocity. Therefore, the force of the globe's comparative weight, (which generates its motion, in the time it describes $d \times \frac{4}{3}$ diam.) : force that generates the same motion, (while it describes $\frac{8}{3}$ diam.)

$\frac{2}{3}$ diam.) : : $\frac{8}{3}$ diam. : $d \times \frac{8}{3}$ diam. :: 1 : d :: den. Fig. of the fluid : density of the globe. Therefore, &c.

[Cor. 3. Pr. 38.] The velocity being given, the motion M is also given; but (by this Prop.) resistance $\times d =$ force, which will generate the motion M (in the time of moving $\frac{8}{3}$ diameter); but this force must be $=$ resistance, which will quite take away the said motion M (in the same time of moving $\frac{8}{3}$ diameter). Now augment the time of moving (preserving the velocity) in any ratio, (let it $= T$.) And this last resistance (which will quite take away the said motion M , in the time T) will be decreased in the same ratio ($= R$). Therefore T and R are also given. Therefore by cor. 7, &c.

[Cor. 4. Prop. 38.] for let $M = \frac{8}{3}$ diameter in the time T ; and because (by sup.) $\frac{tM}{T+t} = \frac{2}{3}M$, therefore $T = t$. Now (by cor. 7. pr. 35.) it will be $(\frac{t}{T} =) 1 : (\log \frac{T+t}{T} \times 2, 3, \text{ &c.} = \log 2 \times 2,$

$3, \text{ &c.} =) , 693 : : \frac{8}{3}$ diameter : $\frac{5.545}{3}$ diameters,

which is less than two diameters. Now that M ought to be $= \frac{8}{3}$ diameter in the time T is plain; for it is required (by cor. 7. pr. 35.) that the resistance R quite takes away M in the time T . But (by this prop. 38.) the resistance ($=$ force required) will just take away the motion M in moving $\frac{8}{3}$ diameter. But if T be any other time than that of moving $\frac{8}{3}$ diameter, then the force to destroy M in that time will not be $=$ resistance, but greater or lesser than it.

[Prop. 40. let the globe, &c.] This is calculated in Prob. XIV. of my book of Fluxions. In cor. 2. of that Prob. G, H and N are found, and thence

Fig. the velocity = $\frac{N - 1}{N + 1} H$, where H is the greatest

velocity the globe can acquire, in falling in the fluid; and the height fallen comes out the same in value with his, but in other symbols; but the one may be easily reduced to the other.

[ib. the numbers in the fourth column, &c.] $2F$ is the space described in the time G with the greatest velocity H , (by the Prob.) therefore, $G : 2F ::$

$P : \frac{2P}{G} F =$ the space described in the time P with the greatest velocity, the numbers in the fourth column.

[Sch. Pr. 40. and any other globe, &c.] that is, the magnitude of any body \propto the excess of its weight in vacuo above that in water. Or $132,8 : 1 ::$ this said excess : magnitude of the globe. Hence, Diameter globe (in Exp. 1.) = $0,84224$.

[ib. Exp. 1. but this space, by reason, &c.] to demonstrate this, it is necessary to premise, that, the greatest velocity wherewith a globe can descend by its comparative weight thro' a fluid in a canal, is that which it may acquire by falling with the same weight, without resistance, and in its fall describing a space which is to $\frac{1}{3}$ diameter as mn^2d , to l^3 . Putting density globe : density fluid : : $d : 1$, and l = orifice of the vessel, $N = l -$ great circle of the globe, $m = l - \frac{1}{2}$ great circle. For (as in cor. 2. Pr. 38.) the space it afterwards describes (in the same time of its fall) = $\frac{mn^2d}{l^3} \times \frac{1}{3}$ diameter; then

the force of the globe's comparative weight (which generates its motion, in the time it describes $\frac{mn^2d}{l^3} \times \frac{1}{3}$ diameter) : force (that generates the same motion in the time it describes $\frac{1}{3}$ diameter) : : $(\frac{1}{3} dia.$

($\frac{1}{3}$ diameter : $\frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter ::) $l^3 : mn^2d$; therefore Fig.

fore (by Prop. 39.) the resistance of the globe = force of the globe's comparative weight (which generates this motion in the time it describes $\frac{mn^2d}{l^3}$

$\times \frac{8}{3}$ diameter, or falls $\frac{mn^2d}{l^3} \times \frac{4}{3}$ diam.) And therefore this force cannot accelerate the globe. Q. E. D.

Now,

By the experiment; $2F = d \times \frac{8}{3}$ diameter; but the space the globe uniformly describes (with the greatest velocity wherewith it can descend in the canal or vessel, and in the time of acquiring it with its compound weight) $= \frac{mn^2d}{l^3} \times \frac{8}{3}$ diameter $= \frac{mn^2}{l^3} \times 2F = \frac{mn^2}{l^3} \times 4.4251$. Also $\sqrt{95,219} :$

$$\sqrt{\frac{mn^2}{l^3}} \times F :: 1'' : \sqrt{\frac{mn^2}{l^3}} \times \sqrt{\frac{F}{95,219}} = \sqrt{\frac{mn^2}{l^3}}$$

$\times G = \sqrt{\frac{mn^2}{l^3}} \times 0.15244$ (by the laws of falling bodies) = time of acquiring the greatest velocity.

And therefore $(\sqrt{\frac{mn^2}{l^3}} \times G (= \text{time}) : \frac{mn^2}{l^3} \times 2F$

(= space) ::) $G : \sqrt{\frac{mn^2}{l^3}} \times 2F :: 4'' : \frac{n}{l} \sqrt{\frac{m}{l}} \times \frac{8F}{G} = (\text{space described in } 4'') = \frac{n}{l} \sqrt{\frac{m}{l}} \times 116$,

1245. Subduct $\frac{mn^2}{l^3} \times 1.3862944F = \frac{mn^2}{l^3} \times$

3,0676. And $\frac{n}{l} \sqrt{\frac{m}{l}} \times 116,1245 - \frac{n}{l} \sqrt{\frac{m}{l}} \times$

3,0676 = space fallen in the fluid, = 116,1245

Fig. — $3,0676 \times \frac{\pi}{l} \sqrt{\frac{m}{l}}$, nearly, as the author makes it.

[ib. ex. 13.] This and the following are upon the same computation as the foregoing experiments in water, to wit, the theory of non-elastic fluids; the reason is, because he considers fluids as elastic, whose parts are discontinued, &c. see Sch. Pr. 35.

[ib. pa. 160. equal to $\frac{1}{4586}$, &c.] This is plain from what follows, afterwards. For let a space : $\frac{2}{3}D :: 860 : 1$; this space $= \frac{6880}{3} D = 2293 D$, and this space the globe moves in the time T , with velocity V ; let therefore $T = 2293$, and $t = \frac{1}{2}$. Then, motion lost $= \frac{\frac{1}{2}V}{2293 + \frac{1}{2}} = \frac{1}{4587} V$; note $t (= \frac{1}{2})$ is not sensibly increased, by the resistance, in moving thro' $\frac{2}{3}D$; and hence I have taken it for the same.

[ib. let D be the, &c.] for, put density globe : density fluid $:: d : 1$; also let the space $\frac{8d}{3}D = S$.

Now by Prop. 38, $d \times$ resistance $=$ force, which (uniformly continued) will quite destroy its motion, in the time it moves $\frac{2}{3}D$; therefore $d \times$ resist. will quite take away its motion, whilst it moves $\frac{2}{3}D$. But (because the force of that resistance and the time of its acting are reciprocally proportional), the resistance the globe meets with will quite take away its motion whilst it moves (with V) $\frac{8d}{3}D = S$. Therefore (by cor. 7.

Pr. 35.) the globe in the time T (of moving S), meets with a resistance R , which will quite destroy its motion V ; therefore in the time t , it will lose

$$\frac{t}{T+t} V, \text{ &c.}$$

S E C T.

SECT. VIII.

[Pr. 42. and because the motion of the waves, &c.] Let the waves A, B, C move from A towards C, and suppose the water to descend from the vertex C into the hollow E, and thence up to B, which it will do supposing the vertices A, B, C to be of equal altitude; also let the water descend from B to D and ascend to A, &c. Now since the water in the vertex B descends towards D, at the same time that other water (that had descended from C to E,) is ascending to B; therefore the water will be lower at the point B and higher at the point d than before; and consequently that the vertex of the wave B will be transferred towards C. Therefore the vertices (and hollows and any correspondent points) of all the waves will move from thence towards C; therefore the waves will be carried on by the deflux of water down CE, BD; and the accumulation thereof, on EB, DA.

[Pr. 43. case 1: at nearly equal distances, &c.] they set forward at equal distances, and continue so, because they move with equal velocity, as appears by (the demonst. of case 1, 2, 3 of) Prop. 48. 45.

[Pr. 46. because the motion of the waves is carried on, &c.] Let A be the vertex of a wave. The fluid at b is more pressed (by reason of a greater depth of incumbent water) than d; therefore that greater pressure causes the water to recede from b in all directions, and to ascend at c; therefore the vertex of the wave is now in c. Also a greater pressure of water propagated from d to f, causes the water to descend at e and ascend at c, therefore the vertex of the wave moves along a, c, e, &c. And this it will always continue to do, if at first any force is supposed to be applied

A C O M M E N T O N

Fig. to the side gba , to hinder the ascent of the water there in like manner as it ascends at c, e ; for then the side gba will be afterwards always descending, and the side ace ascending, without the continuance of such force.

[ib. transverse measure] is measuring along the surface of the water; as is plain from the demonstration.

46. [ib. cor. 2. rather performed in a circle] as in the note on Prop. 42. But if the breadth of the waves be measured, not in the right lines acb , but along the surface of the water (because the deflux of water thro' b, c, a is made in a curve line near a circle), then the time mentioned in the prop. will be very nearly as it is there assigned.

47. [Prop. 47.] fig. 1. pl. 9. is wrong, it should be thus; (and the original is wrong too,) where l, m, n , stand at N, M, L.

[ib.—with those of an oscillating pendulum.]
 [ib. according to the law of a pendulum oscill. for, by Prop. 52. lib. 1. space described by a pendulum, in the time PI; is to a whole vibration; as the time PI, to the time PIS; and to two vibrations, as PI, to PHSP, and the accelerating force of a pendulum, is as its distance from the lowest point. See cor. Pr. 51.

[ib. that is, (because HL — KN is, &c.) that is, HL — KN : HK :: OM : OI.

[Pr. 48. case 1. distances of the pulses, &c.] the distances of the pulses in one medium be = distances of pulses in the other.

[ib. and therefore the pulses; hence it appears that the correspondent parts in both mediums, (vibrate or) go and return together, or in equal times, tho' one makes longer vibrations than the other. But (by supposition) the distances of pulses in both mediums are equal; and the pulses in both

both are translated thro' these equal distances in the Fig. times of the parts of the medium's (vibrating or) going and returning, which times are proved equal; therefore the pulses in both mediums move equal distances in equal times, and therefore are equally swift.

[Pr. 48. case 2. then will their contractions, &c.] be equal, in equal spaces.

[ib. and moreover, &c.] for space \propto square of the time (for these spaces are generated matter

by accelerative forces), and time $\propto \sqrt{\text{space}} \times \sqrt{\text{matter}} \propto \sqrt{\text{space} \times \text{space}} \propto \text{space}$.

[ib. Case 3. the time which is necessary, &c.] for space $\propto \frac{\text{time} \times \text{force}}{\text{matter}} \propto \frac{\text{time} \times \text{force}}{\text{density}}$.

Therefore if the space be given, time $\propto \sqrt{\frac{\text{density}}{\text{force}}}$.

Therefore velocity $\propto \frac{1}{\text{time}} \propto \sqrt{\frac{\text{force}}{\text{density}}}$.

[Prop. 49. — ratio of PO to A conjunctly;]
that is, time $\sqrt{\frac{\text{matter}}{\text{force}}} \propto \frac{1}{\sqrt{\text{force}}}$, when the quantity of matter is given, (this in accelerated motions). Then time of one vibration by that elastic force : time of one vibration of a pendulum PO, by the weight :: $\sqrt{VV} : \sqrt{PO \times A}$, and time of one vibration of PO, by the weight : time of one vibration of A :: $\sqrt{PO} : \sqrt{A}$. Therefore, &c.

[ib. —of the going and returning] of the pendulum PO.

[ib. — of one oscillation composed of the going and returning,] of the pendulum A.

[ib.

A C O M M E N T O N

Fig. [ib. cor. 1. — to its circumference.] For (Mechanics, Prop. 24.) diameter : circumference :: time of fall thro' $\frac{1}{2}$ radius : time of one vibration. Therefore as radius : circumference :: time of fall thro' $\frac{1}{2}$ radius : time of two vibrations.

[ib. cor. 2.] for velocity $\propto \frac{\text{space}}{\text{time}} \propto \frac{A}{\sqrt{A}} \propto \sqrt{A} \propto \sqrt{\frac{e.\text{force}}{\text{density}}}$. By elastic force he means, that which arises by heat, as well as that arising from the compression.

[Prop. 50. Let the number] of the double vibrations, &c.

[ib. Schol. in the subduplicate ratio of the defect of the matter] that is, of the quantity of vapour or watery particles. And (by Prop. 49, cor 2.) the velocity will be increased as (the $\sqrt{\text{density}}$ that is, as) the $\sqrt{\text{quantity}}$ of true air in a given space, is decreased.

[ib. near 100 pulses in] This computation presupposes, that those vibrations of the string are double vibrations, or vibrations composed of its going and return.

S E C T. IX.

[Prop. 51, but the differences of the angular motions, &c.] these are as the difference of the absolute motions round the axis, that is, as the difference of the absolute translations, or as the relative translations (which are those here spoken of) directly, and the distances inversely.

[ib. of quadratures of curves, &c.] Let $SD = x$; $Dd = y$; then $Dd \propto \frac{1}{xx}$ ($= \frac{b}{xx}$) by construction; therefore $y = \frac{b}{xx}$; and $y\dot{x} = b\dot{x} - x\ddot{x}$; and

bx

$$\frac{bx^{-1}}{-1} = \frac{-b}{x} = \text{area}; \text{ therefore area } \propto \frac{1}{x} = DdQ.$$

[ib. cor. 3.] In a given time let the parts 1, 2, 3, 48. of the fluid describe the spaces $1r, 2s, 3t$; take away or add the equal angular motions $1n, 2o, 3p$; so that the spaces nr, os, pt be described in that given time; now whether the points 1, 2, 3 describe the spaces $1r, 2s$, or nr, os ; the translations at the end of the given time will be $= si$; also if 2, 3 describe $2s, 3t$, or os, pt , the translations at the end of the given time will be the same in either case, viz. $= ut$; therefore the translations being equal and the attrition also, the motions will be continued.

[Pr. 52. cor. 8. about any given axis] passing thro' the globe; the relative motion of the parts of the globe and fluid are the same still, therefore, &c.

[ib. cor. 9. semidiameter of the globe;] let BC 49. be a plane, D a point of the vessel, A a point of the globe; then period. time of A : time B :: < velocity B : < velocity A; and therefore time A + time B : time A :: (vel. B + vel. A =) < vel. A from B : < vel. B : time B : time A from the plane BC (to BC again) :: $CB^2 : CA^2$. But time B from DC = time D from BC; therefore, time D (from BC) :: time A (from BC) :: $CB^2 : CA^2$; as it should be by the prop. and cor. 8.

B O O K III.

[**P**ROP. 3. the action of the sun, attracting, &c.]
see cor. 7. Pr. 66. lib. 1, or cor. 14, where
KL is \propto PT nearly, (SK, or ST being given);
and the force TM in its mean quantity, is $=$
PT, nearly.

[Prop. 4. and the space which a heavy body
describes] by falling in $1''$: half the length of the
pendulum ($= \frac{1}{2}$ radius) :: square of $1''$: square of
the time of falling thro' half the pendulum ::
(Mechanics, Prop. 24.) square of the circumference:
square of the diameter.

[ib. pa. 216. line 13, the mean distance of 60
diameters] it should be 60 semidiameters, as it is
in the original.

[Prop. 6. p. 222.—subduplicate of that propor-
tion, as by some computations I have found,] i. e.
as I found by making some calculation.

For let R be the distance of the Sun and Ju-
piter, d the greater force, e the lesser; then to find
the distance x where d may be diminished to e.
Because the forces are reciprocally as the squares
of the distances, say $\frac{1}{R^2} : \frac{1}{x^2} :: d : e :: x^2 : R^2$.

And $x : R :: \sqrt{d} : \sqrt{e}$; therefore at the distance
x, the weight d (of the satellite) would become
equal to e; and the center of their orbits, would
not be in Jupiter, contrary to experience. And
if the force of the satellite was less; by a like
computation, the center of their orbits would be
nearer than Jupiter, also contrary to experience.

[Prop.

[Prop. 8. cor. 1. and $\frac{1}{169282}$ respectively] this Fig.

should be $\frac{1}{196282}$, or rather $\frac{1}{194075}$.

[ib. at the distances 10000, &c.] these are as the diameters; and the apparent diameters are found by astronomical observation; which \times by the proportioned distances, gives the ratio of the real diameters.

[Pr. 8. cor. 4. — of so much the greater density.] This is not generally true, as appears by cor. 2 and 3.

[ib. cor. 3. — truly defined.] This is demonstrated in Gravesand, L. IV. p. 232, 3. See Appendix, p. 20.

[Pr. 10. lose almost a tenth part of its motion.] For by Schol. Pr. 40, p. 160, it is, $1 : 860 : : \frac{8}{3} : 2293.33 +$ space described in the time T ; putting $D = 1$, $V = 1$; and $T : t :: 2293.33 : 2293.33 \frac{t}{T}$ space (uniformly) described in the time t . Then (pa. 161.) $\frac{t}{T} : 2,302$, &c. \times Log.

$$\frac{T+t}{T} :: 2293.33 \frac{t}{T} : 229.5; \text{ therefore } 2293.33$$

$$\times 2.30258 \times \text{Log. } \frac{T+t}{T} = 229.5; \text{ and } \log. \frac{T+t}{T}$$

$$= ,043461; \text{ and therefore } \frac{T+t}{T} = 1.105 + \text{ and}$$

$$\frac{T}{T+t} = \frac{1}{1.105} = \frac{9}{10} + \text{ and } \frac{t}{T+t} = - \frac{1}{10} \text{ nearly.}$$

[ib. or as 75 (0^{12}) to 1 nearly.] In fig. 3. pl. 5. L. II. Let SA = rad. earth = 20949674 feet. AB = 1. AF = 200 miles = 1056000 feet; then, by cor. pr. 22. II. $Aa - Bb : Aa - Ff :: thin : thick$; that is, $AB \times SF : AF \times SB ::$ or $1 : 1005325$

F ::

Fig. :: $t_{bn} : t_{bnz} ::$ (by sch. pr. 86 hyperbola) $\log.$
 $AH - \log. BI : \log. AH - \log. FN$; because
water is 860 times denser than air, and the incum-
bent weight of the atmosphere at the surface of the
earth is about = weight of 33 feet of water;
therefore the weights (and consequently the den-
sities) in A and B, are as 28380 and 28379; whence
 AH or $S_t = 28380$, and BI or $S_n = 28379$; and
 $\log. AH - \log. BI = ,0000152$; $\log. AH =$
 4.4530124 ; therefore $1 : 1005325 :: ,0000152 :$
 $4.4530124 - \log. FN$; and $\log. FN = -11,1730$.
And therefore $FN = ,000000000015$, and density
in F : density in A :: 15 : $2838000000000000 ::$
or as 1 : to 1800000000000000 , which is less than
1 to 750000000000000 ; but the computation will
vary as the radius of the earth, the density of wa-
ter and air at the earth, is supposed to vary.

[ib. and hence the, &c.] as before (by Pr. 40.
Sch.) since density of water : to density of air 200
miles from the earth :: 645 (0^{14}) : 1. Space de-
scribed in time $T = 172 (0^{15})$. And $\frac{t}{T} : \log.$

$\frac{T+t}{T} \times 2.302$, &c. :: $172 (0^{15}) \times \frac{t}{T}$ (= space
described in time t) : $2792250000 =$ space Jupiter
describes in 1000000 years. Therefore $\log. \frac{T+t}{T}$
= $,0000000705$. And $\frac{T+t}{T} = 1.000000017$.

And $\frac{T}{T+t} = ,999999986$. And $\frac{t}{T+t} =$
 $,000000014$, which is less than $\frac{1}{1000000}$.

[Pr. 13. are almost as 16, &c.] for (by this and
cor. 2. Pr. 8.) accelerative force of Jupiter towards
Saturn :

Saturn : accelerative force of Jupiter towards the Fig.

$$\text{Sun} :: 25 \times \frac{1}{3021} :: 16 \times 1 :: 81 :: \frac{16 \times 81 \times 3021}{25}$$

[Pr. 14. Sch. And hence we may find, &c.] For by cor. 7. prop. 66. I. The apsides of their orbits move in consequentia. And by cor. 16, of the same, the motion of the apsides of the body P (plate 21. fig. 2.) will be as the periodical time of P directly, and the square of the periodical time of T inversely, that is, as the periodical time of P (because the periodic time of T, round S, or of the Sun round Jupiter or Saturn, or (which is the same thing) of Jupiter or Saturn round the Sun, is given); that is (by prop. 15. I.) in the sesquiplicate ratio of their distances PT.

[Pr. 19. in the duplicate proportion of the rad. to the cosine of latt.] For (by cor. 3. pr. 4. L. I.) centrifugal force at the equator : centrifugal force at a from the axis ($= ab$) :: rad. : cos. lat. :: $r : c$. Also the force directly from the axis : force directly from the earth :: $ab : bc :: r : c$. Therefore, ex equo, centrifugal force at the equator : centrifugal force directly from the earth, at $a :: rr : cc$.

[ib. but by computation (from] Let $GC = t = 51$
 101 . $BP = C = 100$. $PE = x$. Then will $ED^2 = \frac{tt}{cc} \times cx - xx$. And $ER^2 = PD^2 = \frac{ttx}{c} - \frac{tt}{cc} x^2$

+ x^2 . And $ER = \sqrt{\frac{tt}{c} x + \frac{cc - tt}{cc} x^2} =$ (suppose to) $\sqrt{r^2 x - sx^2}$. Whence the fluxion of the area PQRE is $= \frac{r^2 x - sx^2}{2} \dot{x} = rx^2 \dot{x} - \frac{sx^3}{2r} \dot{x}$
 $- \frac{s^2 x^2}{2.4r^3} \dot{x} - \frac{3s^3 x^2}{2.4.6r^3} \dot{x} - \frac{3 \cdot 5 s^4 x^2}{2.4.6.8r^5} \dot{x}$ &c. And

the area PQRE = $\frac{2}{3} rx^3 - \frac{sx^2}{5r} - \frac{s^2 x^2}{7.4r^3} - \frac{3s^3 x^2}{9.4.6r^5}$
 F 2

A C O M M E N T O N

$$\text{Fig. } - \frac{3.5.s^4x^{11}}{11.4.6.8r^7} - \&c. = \frac{2}{3}rx^3 - \frac{3.sx}{2.5r^2} A + \frac{5.sx}{7.4r^2}$$

$$B + \frac{7.3.sx}{9.6r^2} C + \frac{9.5.sx}{11.8.r^2} D + \&c. = (\text{putting } \frac{sx}{rr} \\ = q) \frac{2}{3}rx^3 - \frac{3}{10}Aq + \frac{7}{10}Bq + \frac{5}{10}Cq + \frac{5}{10}Dq \\ + \&c. = (\text{when } x = c = 100; \text{ and } s \text{ being } = \\ \frac{cc - tt}{cc} = ,0201. \text{ And } r^2 = \frac{tt}{c} = 102,01. =) \text{ to}$$

$$6693.39. \text{ And PBM} = 5000. \text{ Whence PQRM} \\ = 1693.39. \text{ Therefore by cor. 2. pr. 91. I. force} \\ \text{of the spheroid : force of the sphere :: (when P} \\ \text{and A coincide) AS - \frac{AS \times PQRM}{CS^2} : \frac{AS}{3} :: 1 -$$

$$\frac{PQRM}{CS^2} : \frac{1}{3} :: 1 - \frac{4 \times 1693.39}{10201} : \frac{1}{3} :: 1,008 : 1 ::$$

126 : 125.

[ib. p. 243. if the density, &c.] this will appear
by considering what went before, and cor. 3. pr.
91. I.

[ib. but if the diurnal, &c.] for the (centripetal
or) centrifugal force, is as the square of the velo-
city when the rad. is given.

[ib. 244. — augmented in proportion as the force
of gravity is diminished.] for, reviewing the former
calculation ; if the density of the earth were greater
than it is ; the force of gravity, to the centrifugal
force, would be greater than in that proportion of
289 to 1 ; (and the two diameters would be in a
less proportion than 289 to 288 ; and also the dia-
meters would be in a less proportion than 230 to
229;) consequently the ratio of the difference of
the diameters to either, decreases as the gravity in-
creases. And on the contrary, that ratio increases
as the gravity decreases. And upon this depends

the foregoing proportion, as $\frac{4}{505} : \frac{1}{100} :: \frac{1}{289} :$

$\frac{1}{229}$

{ib.

[ib. line 11.] read $\frac{29}{5} \times \frac{400}{94\frac{1}{2}} \times \frac{1}{229}$ to 1. In Fig.
the Original, 2d edit. it is $\frac{29}{5} \times \frac{5}{1} \times \frac{1}{229}$ to 1,
or as 1 to 8.

[Prop. 20. Whence arises this Theorem] for let 52.
 $nkyn$ be a circle circumscribed round the ellipsis
 $abyl$. Then $kbm : fbc (=) abd :: kt^2 : fg^2$. And
 $ab = \frac{kbm \times fg^2}{kt^2 \times bd}$. But because kbm and kt are given,
and $bd =$ (nearly to) $2kt$ which is given. Therefore
 $ab \propto fg^2$; but the difference between the
weight at n and a , is as ($at - bt =$) ab . Therefore
the increase of weight is as fg^2 , or the square
of the sine of lat. at a , nearly, or (by trigonomo-
try) as the verf. of $2an$.

[ib. and the arcs of the degrees, &c.] The
length of a degree is OC rad. of curvature, and
in the points n and b , are (by ex. 1. pr. 19. Sect.
II. Curve lines) as the parameters of ny and bl ,
that is, as bb^3 to ny^3 , or 229^3 to 230^3 , that is, as
 56637 to 57382 , as is inserted in the Table,
page 247.

But to find the rad. curvature at b ; we have (by
Ex. 2. Prob. 5. Sect. II. Fluxions) $mH = y +$
 $\frac{4 \times a - b}{abb} y^3$; and therefore (putting s for the sine
of the angle ntb); rad. curvature mc at $b =$
 $\frac{y}{s} + \frac{4 \times a - b}{abbs} y^3 =$ (because $\frac{y}{s}$ is a given ratio,)
 $R + \frac{4 \times a - b}{abb} Ryy$, very near, R being the radius
of curvature in n . And therefore the increase of
this radius, which is as the increase of the degrees,
is as yy or ss nearly. And here $a - b$ being very
small, $\frac{4 \times a - b}{abb} R$, is an extremely small quantity.

A C O M M E N T O N

Fig. [Pr. 23. — which I cannot here descend to explain.] Since the moon's orbit is more excentric than those of Jupiter's satellites, the motion of the moon's apogee will be greater in proportion, than the motion of the apogee in any satellite.

53. [Pr. 27. The area, &c.] For the moment of the area is $\propto cb \times ad$. But $ad \propto$ angle $bcd \times cb =$ horary motion $\times cb$. Therefore, area $\propto cb^2 \times$ horary motion.

[Pr. 28. But the attraction of the moon ;] for let A = attraction of P towards T. Let the attraction or force LS be resolved into the two, LM, SM ; the first acting towards M the latter towards S. Then the whole attraction of P towards T is $A = ST + LM - TM$. Let $A - ST = F$. Then the attraction of P towards T in the syzygies is $F - 2AT$; and in the quadratures $F + CT$. By prop. 25; $F : ML :: 178725 : 1000$. And the forces $2AT$, and CT , will be to each other as 2000 AT, and 1000 CT, that is, as $\frac{2000}{CT \times N}$, and

$\frac{1000}{AT \times N}$. And the force F in the syzygies and quadratures will be as 178725 directly, and the square of the distance from T reciprocally. Therefore, &c. (Note, $+ \frac{2000}{CT \times N}$ is false printed for $- \frac{2000}{CT \times N}$, in line 10, pa. 268.)

{ib. — Quadrature in C; or, which —] for $CTP : CTp ::$ angular motion of the moon from the sun's quadrature : to its angular motion from the fixt point C, that is (because the given circumference, or one revolution, is to be described in either case) : : periodic revolution : synodic revolution.

54. {ib. But by computation I find,] with the radii TA , TC , describe the circles Aic , Cic . And let AZ

AZCz, touch the ellipsis in A and C. And take AlFig. to Ac, and Ci to Cc, as $\angle \text{CTP}$ to CTp (fig. 5. 54. pl. 10.) Now in order to determine the curvature of the orbit, suppose the point a to coincide with A, and draw the parts of the orbit ARO, Cro, which will pass between the ellipsis and circle; for TO or TE is less than TL, and greater than (TA or) Tc. And To (that is Te) is less than TC or Tc, and greater than TL. Draw the lines TB, Tb, so that $AB = Cb$. And let the points A, I, C, and also C, i, c coincide; and the curvatures of the ellipsis, orbit, and circles at the points A, C, will be respectively as BE, BR, BI, and be , br , bi . And the difference of curvatures of the ellipsis and circles, and of the orbit and circles in A, C, will be as EI, RI, and ei , ri , respectively. But RI : EI or OC (because TE = TO, and TI = Tc ::) $\text{AI}^2 : \text{Ac}^2$ (Lem. 11. I.) :: $\text{CTP}^2 : \text{CTp}^2$. Whence $RI = EI \times \frac{\text{CTP}^2}{\text{CTp}^2} = BI - BE \times \frac{\text{CTP}^2}{\text{CTp}^2}$. Also, $ri : oc$ or ei , or $be - bi :: Ci^2 : Cc^2 :: \text{CTP}^2 : \text{CTp}^2$. Whence $ri = \overline{be - bi} \times \frac{\text{CTP}^2}{\text{CTp}^2}$. But in the \odot TA; BI, AI or AB, and $2AT$ are \perp . And in the \odot TC; bi, Ci or Cb, $2CT$ are \perp ; whence $BI = \frac{AB^2}{2AT}$; and $bi = \frac{Cb^2}{2CT}$. Also in the ellipsis, $BE \times 2AT : BA^2 :: AT^2 : TC^2$. And $be \times 2CT : bC^2 :: CT^2 : AT^2$. Whence $BE = \frac{BA^2 \times AT}{2CT}$. And $be = \frac{bC^2 \times CT}{2AT}$. Now from hence, the curvature of the orbit at a : curvature of the orbit at C :: BR or BI - IR : br or bi + ir :: $\frac{AB^2}{2AT} - \frac{BI - BE}{2AT}$.

$$\begin{aligned}
 \text{Fig. } & \times \frac{\text{CTP}^2}{\text{CTP}^2} : \frac{\text{Cb}^2}{2\text{CT}^2} + \frac{be - bi}{2\text{AT}} \times \frac{\text{CTP}^2}{\text{CTP}^2} :: \frac{\text{AB}^2}{2\text{AT}} \times \\
 & \frac{\text{CTP}^2}{\text{CTP}^2} - \text{BI} + \text{BE} : \frac{\text{Cb}^2}{2\text{CT}^2} \times \frac{\text{CTP}^2}{\text{CTP}^2} + \frac{be - bi}{2\text{AT}} :: \\
 & \frac{\text{AB}^2}{2\text{AT}} \times \frac{\text{CTP}^2}{\text{CTP}^2} - \frac{\text{AB}^2}{2\text{AT}} + \frac{\text{BA}^2 \times \text{AT}}{2\text{TC}^2} : \frac{\text{Cb}^2}{2\text{CT}^2} \times \\
 & \frac{\text{CTP}^2}{\text{CTP}^2} + \frac{bC^2 \times \text{CT}}{2\text{AT}^2} - \frac{\text{Cb}^2}{2\text{CT}} : \frac{\text{CTP}^2}{\text{AT} \times \text{CTP}^2} - \\
 & \frac{1}{\text{AT}} + \frac{\text{AT}}{\text{TC}^2} : \frac{\text{CTP}^2}{\text{CTP}^2 \times \text{CT}^2} + \frac{\text{CT}}{\text{AT}^2} - \frac{1}{\text{CT}} :: \\
 & \frac{\text{CTP}^2 - \text{CTP}^2}{\text{AT} \times \text{CTP}^2} + \frac{\text{AT}}{\text{TC}^2} : \frac{\text{CTP}^2 - \text{CTP}^2}{\text{CT} \times \text{CTP}^2} + \frac{\text{CT}}{\text{AT}^2} \\
 & :: \text{AT}^2 + \frac{\text{CTP}^2 - \text{CTP}^2}{\text{CTP}^2} \times \text{AT} \times \text{CT}^2 : \text{CT}^3 + \\
 & \frac{\text{CTP}^2 - \text{CTP}^2}{\text{CTP}^2} \times \text{CT} \times \text{AT}^2.
 \end{aligned}$$

55. [Pr. 29. The tangent of the angle CTP, &c.] On the axis DC describe the circle CdD, and let the line Tb revolve uniformly to d, in the same time that TP revolves to A, and describes areas proportional to the times in the ellipsis CAD. Draw be || to dT, and draw Tb, TP. Since the circle and ellipsis are described in equal times, parts proportional to the whole will be described in any equal times; now area ellipsis : area circle :: TA : Ta (= TC) :: eP : eb :: (ePC + ePT =) TPC : (ebC + ebT =) TbC. Therefore the point in the circle is at b, when the point in the ellipsis is at P. But TA : (Td =) TC :: eP : eb :: tangent < eTP : tangent < eTb (= < of the mean motion.)

[ib. Square of the sine of the angle CTP.] These things are plain from prop. 26, where Pd (fig. 4.) is the excess of the moment, and is as $\frac{\text{PK}^2}{\text{PT}}$.

56. [ib. which we may effect, &c.] Let A be the area described by the moon in any time, arch CP

B. III. THE PRINCIPIA. 89

$CP = z$, $r = \text{radius } TC$. $PK = y$, $TK = u$, Fig.
 $t = \tan. PTC$. Then $A = \frac{yyz}{r} = y\dot{x}$, and $A = 56.$

Fl : $y\dot{x} = \text{area } CPK$. But TPC expresses the mean motion, and therefore TPK is the equation, which is as $TK \times PK$ or uy .

But by the nature of the circle, $\dot{z} = \frac{rrt}{rr+tt}$, and since t is in a given ratio to t , (or as .3123 to 69,) therefore \dot{z} is as $\frac{rrt}{rr+tt}$ or $\frac{t}{\sqrt{rr+tt}} \times TK$ or $y \times TK$, or as uy . Therefore decreasing the tangent in the subduplicate ratio of 11073 to 10973, or in the simple ratio of 69 to 68, 6877 ; accelerates the area in proportion of PK^2 , as it ought to do.

[ib.—in a proportion compounded of the duplicate, &c.] for (by cor. 16. pr. 66. L. I.) all angular errors are as the square of the time of the moon's revolution, directly, and the square of the time of the earth's revolution inversly ; that is (by pr. 15. L. I.) as the square of the time of the moon's revolution directly, and the cube of the earth's distance from the sun inversly.

[Pr. 30. And this force by prop. 25, is, &c.] The force $3PK$: force $ML :: 3IT : PT$ (for these are the same) and force ML : centripetal force by which the moon revolves, &c. :: 1 : $178\frac{2}{4}\frac{9}{4}$ (by pr. 25.) therefore *ex equo*, force $3PK$: centripetal force the moon revolves with :: $3IT \times 1 : PT \times 178\frac{2}{4}\frac{9}{4}$. Or as $IT : \text{rad.} \times 59,575$.

[ib. the half of which the moon] it should be, which the moon, by the action of the said force, as it is in the first edition.

[ib. And the angle PTM is equal to the angle, 57. &c.] for in this case LM is perpendicular to MP . Let PR be a tangent to the point K ; then the triangle

Fig. angle RPT is a right one; and angle RPM = 57. angle PTM. And angle LPM : < (RPM =) PTM (when the radius is PM) :: LM : RM :: force producing LM : force producing RM :: 1 : 59,575.

[Pr. 31. cor. and the decrement is to the remaining motion as 100, &c.] The motion of the nodes in the octants : motion in the syzigies :: 11073² : 11023² :: (because 11073, 11023, 10973 are in arithmetical, and nearly in $\frac{1}{2}$ progression) 11073 : 10973; and decrement : remaining motion :: (11073 — 10973 =) 100 : 10973.

58. [ib. but the decrement] for the decrements are as the forces and times, or as the whole motions and times; that is, decrement at H (or increment at b) : decrement at A :: mot. at H, and time : mot. at A, and time :: (that is, by what went before) as mot. at H $\times \frac{yy}{rr} - \frac{1}{2}rr$: mot. at A $\times rr - \frac{1}{2}rr$ (or mot. at A $\times \frac{1}{2}rr$.) This reasoning being very obscure, you will find the motion of the nodes clearly investigated, in (Prop. V. Sect. VI. and cors. of) my Astronomy.

[Prop. 32, it is drawn back again] that is, supposing the place of the node given.

[Prop. 32. Now the area of the semicircle] For let NT = 1, AZ = y, TZ = x. Then eZ = $\frac{y^3}{9.0827 + yy}$, and Ae (AZ — eZ) = $\frac{9.0827y}{10.0827 + yy}$.

But $yy = 1 - xx$, therefore Ae = $\frac{9.0827\sqrt{1 - xx}}{10.0827 - xx}$

$$= \frac{\overline{b-1}\sqrt{1-xx}}{b-xx} = \overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$$

&c. \times into $\sqrt{1-xx}$, putting $b = 10.08276$, and the fluxion area NAe = $\overline{b-1} \times \frac{1}{b} + \frac{xx}{bb} + \frac{x^4}{b^3}$, &c.

B.III. THE PRINCIPIA. 91

&c. $\times \sqrt{1 - xx}$; whose fluent (by form 17. Flux. Fig. ions) $= \frac{1}{b} + \frac{1}{4bb} + \frac{1}{8b^3}$ &c. $\times \sqrt{b-1}$. ϕ (putting $\phi = \text{quadrant or } \frac{NA_n}{2}$) $= .92435 \phi$, therefore half the area $NFn = .07565\phi$. And the area ϕ (or quadrant) is to the area $\frac{NFn}{2}$; or the semicircle, to the whole area NFn ; as 1 to .07565, or as 793 to 60.

[Prop. 33. will nearly agree with] let $N = 19$
 $49 3 55$, $p = \text{periphery of } NA_n$ or DFB , E the equation, $S = S.2NA$.

Then NAZ represents the mean motion of the nodes for the arch NA , as N does for the whole circle; and NAZ is the true motion; and ATZ is the difference proportional to the equation.

Divide by $\frac{1}{2}r$, and then $p : \frac{xy}{r} :: N : E$, or $p : \frac{2xy}{r}$:: $\frac{1}{2}N : E$, that is, $p : S :: \frac{1}{2}N : E$; whence if $BF = 2NA$, then (by construction) $AD : CD :: p : \frac{1}{2}N :: S : E$. Suppose S an arch in DF , then $360 : \frac{1}{2}N :: \text{degrees in } S : E$; but deg. in S (in the circle DF) : degrees in DG (in the circle DG) :: $AD : CD$: (by construction) $360 : \frac{1}{2}N$ in degrees :: degrees in $S : E$; whence the degrees in $DG = E$; or the angle $DAG = \text{equation}$.

[ib. cor] for in the syzygies the areas ANZ and ENZ vanish; and NA is 90° , when they are in the quadratures; and when in the octants, the sine of $BF = S.90 = 1$; therefore,

38.3	—	1.58319
Rad.	—	10.
1	—	○
—		—
$S. < A (1^\circ 30')$		8.41631

[Pr.

A C O M M E N T O N

Fig. [Pr. 2. pa. 291, and by the demonstration] the angle ATN will be the distance from the nodes true place, by construction. Also the mean motion of the sun, in the time NA : mean motion of the node from the sun, in the same time :: (mean annual motion of the sun : mean annual motion of the node from the sun :: (by Pr. 1.) area of the ellipsis : area of the circle ::) TBN : TFN ; but TBN was the mean motion of the sun in the time NA ; and therefore TFN is the mean motion of the node from the sun in that time NA ; and FTN the angle of the mean motion, or the distance of the sun from the mean plane of the node.

59. [ib. cor.] for let $atn = \pi tb = 45^\circ$, and $ft \perp$ to at ; then fc is the sine of fta , the Δ 's fcb and btb are similar; therefore $fc : fb :: bt : bb$; and by composition, $fb : ct$, or $at :: bf : fc$; and by inversion fc is to at or $kt :: fb : fb$ or $by :: kH : Hm$ or $tk + TH$.

[ib. but the sine of] draw $oz \perp$ to it, then oz is to fc in a ratio compounded of (or to fb or of) os to fg , and (the sine of orz or trs to the sine of fbc or tbg , that is, of) ts to tg ; that is, in the compound ratio of $os \times ts$ to $fg \times tg$, that is (because rad. : cosine :: 2^{ce} sine : sine of the double arch, and therefore the rectangle of the sine and cosine being as the sine of the double arch) as sine of $2atn$, to sine of $2ftn$ or radius.

[ib. Sch.] for $TS : (TS + SK) = TK :: 360^\circ : (360^\circ + 39^\circ, 6355) = 399,6355 :: 9,0827646 : 10,0827646$. Therefore $TH : TK :: \sqrt{9,0827646} : \sqrt{10,08}$, &c. And therefore $TH : HK :: 18,6524761 : 1 :: TS : SH ::$ mean motion of the sun : mean motion of the node $= 19^\circ 18' 1'' 23'''$.

Lastly, $TK + TH (= 38,22428 : KH (= 1)) ::$ rad. : sine of $1^\circ 29' 57''$ (by cor.)

[Pr. 34. cor. 2. with due regard] for whilst p moves from Q to F, the sum of the areas is comprehended

prehended by TH produced, FQ, and tangent to Fig. Q; but in moving from F to q, the line Hp falls on the other side of the circumference, and the sum of these areas is that comprehended by FT produced, Fq, and the tangent to q; and the difference of these areas is the semicircle, generated in the time of describing QAq; and in the time of describing the whole circumference, the whole circle will be generated.

[ib. cor. 4. that is, as the diameter] Suppose QP to be the double distance of the moon from the quadratures; MK the sine thereof. Then Kk : Mp :: MK : radius. And the sum of all the Kk's, or the diameter : sum of all the Mp's or QAq :: sum of all the fines : sum of as many radii, or half as many diameters. And therefore (multiplying the antecedents by $\frac{Pp}{PG}$, and the consequents by 2)

the sum of all the fines $\times \frac{Pp}{PG}$: sum of as many diameters :: diameter $\times \frac{Pp}{PG}$: whole circumference.

[Prop. 35. by the same increments as the fine of inclination doth, by cor. 3.] For if AEG be double the distance of the nodes from the quadratures, GED will be double the distance of the nodes from the sun; and they both have the same fine.

[ib. Schol.] Here are several particulars about the moon's motion barely laid down; many of them being taken only from observations.

[ib. pa. 299. The force of this action is greater] It is greater the nearer it is, and attracts the moon from the earth, and so dilates the orbit the more. But the further off, the less force, and the less the moon is drawn from the earth. And at a greater distance

Fig. distance she moves slower; and at a less distance, faster.

[ib. further I found that the apogee] by cor. 14. prop. 16. B. I. And if a be the sun's mean distance, and $a+x$ any other distance, then the motion will be as $\frac{1}{a+x^2}$ or $\frac{1}{a^2} - \frac{3x}{a^4}$, or $\frac{1}{a^2} - \frac{x}{a^2} - \frac{3x}{a^4}$, and the equations, as $\frac{x}{a}$, or as x .

The motion of the sun, that is the angular motion, is reciprocally as the square of the distance.

[ib. but if the motion,] let v be the velocity, m the mean motion of the sun. Then if v be as

$\frac{1}{a+x^2}$, or $\frac{1}{aa} - \frac{2x}{a^3} = m - \frac{2x}{a} m$; then the equa-

tion is $\frac{2x}{a} m$. But if v be as $\frac{1}{a+x^2}$ or $\frac{1}{a^2} - \frac{3x}{a^4}$

$= m - \frac{3x}{a} m$, then the equation is $\frac{3x}{a} m$. And the

first equation to the second, is as $\frac{2x}{a} m$ to $\frac{3x}{a} m$,

or as 2 to 3. And hence the greatest equation of the apogee and nodes (sum of all the $\frac{3x}{a} m$): their

mean motion (m) : : $2^\circ 54' 30''$: sun's mean motion; the greatest equations being as the mean motions.

[ib. pa. 300. By the theory of gravity I likewise found] For then the moon being nearer the sun (in all points of its orbit taken together), the sun's force must be greater.

[ib. p. 301. By the same theory of gravity the action,] For when the sun is in the line of the nodes, its whole force is exerted in moving the moon, and none in moving the plane of its orbit.

[ib.]

[ib. By the same theory of gravity, the moon's Fig.
By cor. 8. prop. 66. B. I.

[Pr. 36, in other positions of the sun,] by the reasoning in cor. 19. pr. 66. L. I. depression of the water at P : ascent at P in the direction PH :: (LM : TM ::) PT : 3PK; and ascent at P in direction PH : ascent at P in direction TP :: (rad. : S.PTK ::) PT : PK; therefore (*ex equo*) depression at P : perpendicular ascent there :: (PT² : 3PK² :: $\frac{1}{3}$ PT² : PK²; but the force PT that depresses P is given, therefore the force to raise the waters at P, is always as PK², that is, as the versed sine of $2PTC$ or twice the sun's altitude.

Also the effects of these forces (by cor. 14. pr. 66. L. I.) at different distances from the sun are reciprocally as the cubes of these distances; and therefore,

Cor. 1. The sun raises the water under it to one Paris foot and $11\frac{1}{3}$ inches.

[Pr. 37, but because of the reciprocation] see pr. 24.

[ib. and the sun's force in—] let A be the position of the moon, and ATP = angle the sun and moon makes at the earth's center; and let PT represent the force of the sun, which divide into the forces PK, KT, PK acting in the direction of the moon increases her force, KT acting in a different direction, diminishes it; and therefore PK — KT (or the difference of the sine and cosine of the angle ATP) is the absolute increase of the moon's force; but this differs not much from the cosine of double the $\angle ATP$, (in an angle of $18^{\circ}\frac{1}{2}$ the difference = ,6310189) and in the arches $0^{\circ} 45^{\circ} 90^{\circ}$ they perfectly agree. Or thus,

Let qc = moon's force, ca = sun's force. If the sun is in n , let $\angle ace = 2 \angle acn$, then qe or qb = sum

Fig. sum of the forces, $cb =$ sun's force, in that position
 62. $= \text{cos. of } 2 < acn.$

[ib. But further, the force of the moon] for let PT be the whole force of the luminary in the place P (or in D) the other side of (the earth or) its parallel; the force DT = the two forces DK, KT. When the luminary is in D, the force to move the water in the direction TP, is TK; and the difference of these, or (TP — TK =) PK is the difference of the forces in the points P, D; or the absolute force to move the sea at the lat. CTP. But PK is ∞ □ fine of ATP, or as □ cos. CTP.

Then since $L + S$ is the whole force when the moon is near the syzygies, and $L - S$ in the quadratures. Therefore $0,8570327 L$ must be substituted for L only in the quantity $L - S$, which represents her force in the quadratures.

[ib. from whence we have] also $1,017522 L$ must be substituted for L, in $L + S$, which represents her force in the syzygies; and $0,9830427 L$ for L, in $L - S$, when she is in the quadratures; for these represent her attractive forces in the syzygies and quadratures.

[Pr. 38. as the accelerative gravity] for if the diameters be given, the forces to raise the water will be ∞ absolute forces; and if the absolute forces are given the spheres will be reduced into similar spheroids, for the perturbating forces will be as the diameters (for the perturbating force is

∞ PT fig. pr. 25.) Therefore if neither be given, the forces to raise the water in the moon and earth, are in the compound ratio of the earth to the moon, and the moon's diameter to the earth's diameter; or as 1090 to :00; therefore the water rises to 93 $\frac{3}{4}$ feet in the moon.

No notice is taken here, of the tides being less by reason of the moon's motion round the earth, than if she stood still in a given position. Since the

the motion of the tides reaches 90° for every ebb-Fig.
bing and flowing; it would seem, that the said
motion could not be propagated thro' 90° to its
greatest height, in the time the moon stays in the
meridian. But (by a computation from pr. 44. II.)
it appears, that water will oscillate in a cahal of 90°
in length in $\frac{2}{3}$ of an hour; and therefore in that
time the tides will be propagated to that distance,
and rise to their highest; but the position of the moon
in that time is not sensibly altered; and therefore
the tides would rise no higher, tho' she were al-
ways to stand in a given position. But this sup-
poses that there is depth enough of sea, to supply
water sufficient for the purpose.

[Lem. 1. to Pr. 39. to recede towards this side
and that side] that is, all the parts of AC to act
in a direction towards the sun; the parts of CE,
directly from it.

[Lem. 2. The matter in the circumference of
every circle IK] by this is meant the circular ring
(in the plane of the circle IK) comprehended be-
tween the surfaces of the sphere and spheroid.

[Lem. 3. about the same axe] he means about
its diameter.

[ib.] I. Let ABD be a spherical surface, and 63.
AEFD a circumscribing cylindric surface, and
their common thickness the infinitely small line Bb
or Aa. Divide AC into an infinite number of equal
parts = in = \dot{x} , and erect the planes id, ne; then
the surfaces of the sphere and cylinder compre-
hended thereby are equal. Now, flux. mot. cyl.
surface : flux. mot. sph. surface :: (as their matter
 \times velocity, and the matter is given, and therefore,
as) CB \times \dot{x} , or id \times \dot{x} : uf \times \dot{x} . And mot. cyl.
surface : mot. sph. surface :: as all the CB \times \dot{x} :
to all the uf \times \dot{x} :: AD² : area of the circle ABDG.

Again. Let ABD be a sphere, and AEFD its 64.
circumscribing cylinder; ECA a cone, all revolv-

G ing

Fig. ing round AD. Divide BC into an infinite number of equal parts $= bd = \dot{x}$. Thro' the points draw spherical surfaces bgd concentric to ABD, and cylindric surfaces bf terminated at the line EC. Then (by what went before) mot. cyl. surface bdf : mot. sph. surface $b dg :: AD^2 : \text{circle } ABDG$. Therefore mot. all cyl. surfaces, or the solid EBC : mot. all the sph. surfaces, or the sphere : in the same given ratio of AD^2 : to circle $ABDG$.

65. Further, flux. mot. cyl. round its axis \propto flux. mot. \times velocity, that is $\propto \dot{e}^2 \times \dot{x}$, or $x^2 \dot{x}$. And the whole motion is $\propto \frac{x^3}{3}$. And therefore the motion of a cylinder round its axis \propto length \times cube of its diameter, \times density; or as its weight \times diameter.

66. Lastly, let EBCA be a cylinder, and ECA a cone. Then flux. cyl. mot. : flux. con. mot. :: $AE^3 \times \dot{x} : ba^3 \times \dot{x} :: CA^3 \times \dot{x} : Cb^3 \times \dot{x}$, or as $r^3 \dot{x} : a^3 \dot{x}$. Mot. cyl. round its axis : mot. cone round its axis :: $r^3 x : \frac{x^4}{4} ::$ (when $x = r = CA$)

$4 : 1$. Therefore, mot. cyl. : mot. fig. EBC (\equiv difference of mot. of cylinder and cone) :: $4 : (4 - 1 =) 3$. hence it follows,

Mot. cyl. : mot. sol. EBC :: $4 : 3$

Mot. sol. EBC : mot. sphere :: diameter \square : circle,
Therefore, *ex equo*,

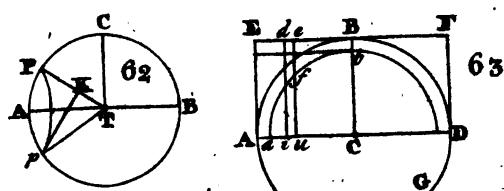
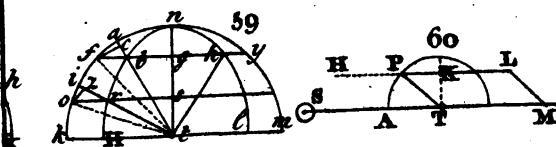
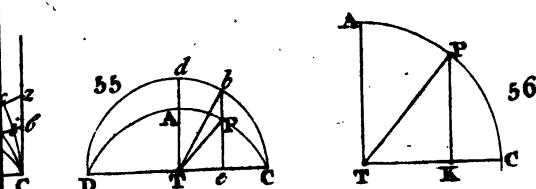
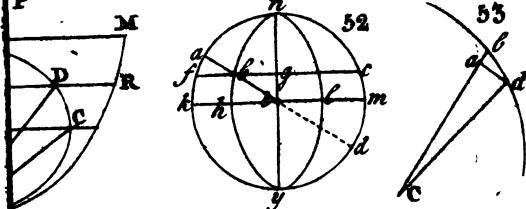
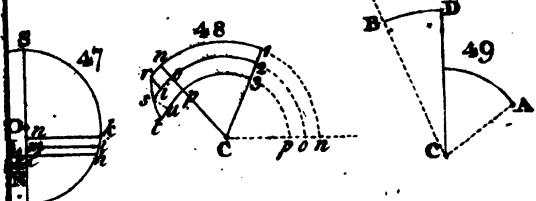
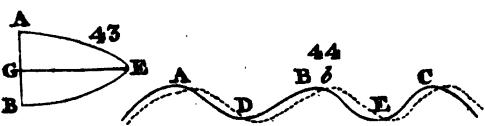
Mot. cylinder : mot. of sphere :: $4 \text{ diam. } \square : 3 \text{ circle}$.

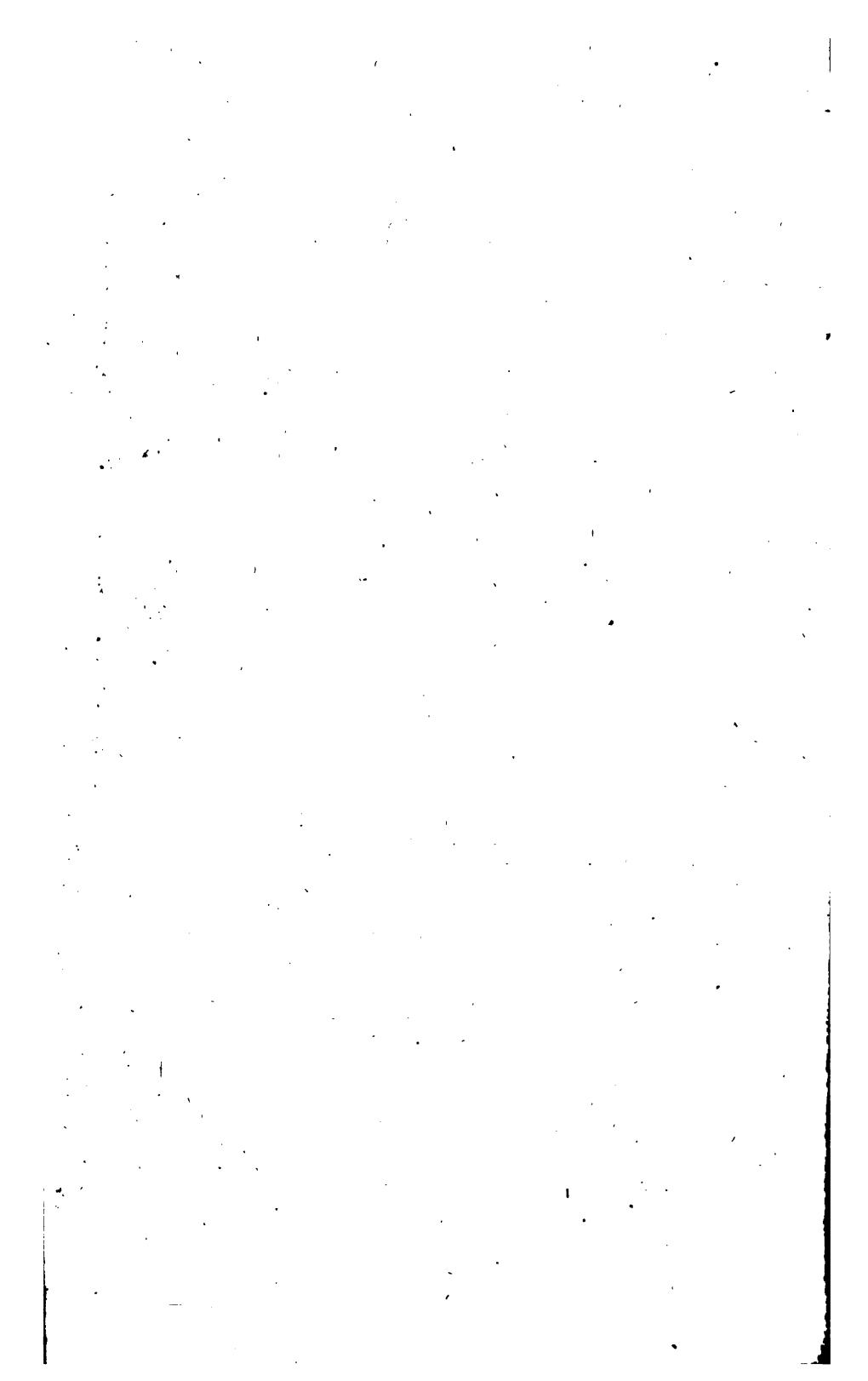
Or,

Mot. sphere : mot. cylinder :: $3 \text{ circles} : 4 \text{ diam. square}$.

67. II. Flux. mot. cyl. : flux. mot. of a ring (or periphery) of the same diameter and matter :: (as velocity \times flux. matter, as) $x^2 \dot{x} : rx\dot{x}$. And mot. cyl. : mot. ring :: $\frac{x^3}{3} : \frac{rx^2}{2} ::$ (when $x = r$) $2 : 3$.

And





B. III. THE PRINCIPIA. 99

And mot. cyl : mot. any ring :: 2 cylinders : 3 Fig. rings.

III. Let $ED = z$; and let \dot{z} be given; then 68.
 $y\dot{z} = r\dot{x}$. Now flux. mot. ring round C : flux.
 mot. ring round AB :: $r\dot{z}$: $y\dot{z}$ or $r\dot{x}$. Therefore
 mot. ring round C : mot. ring round AB : $r\dot{z}$: $r\dot{x}$::
 (when $x = r$) AD : AC :: circumference : 2 dia-
 meters.

Therefore from the 1, 2, 3 Art it follows, *ex equo*, that the motion of a sphere : motion of a ring round their axes :: as the matter in the cylinder and ring, and $6 \times$ circumference \times circle : 24 diam. cube :: or as square of the quadrant arch : to square of the diameter :: that is, as matter in the cylinder to matter in the ring, and 61685 : to 100000 ; or as the matter in the sphere to the matter in the ring, and 925275 , to 1000000 .

[Pr. 39. (for the reasons above explained),] for (by cor. 2. pr. 30.) mean horary motion of the nodes : $16'' 35''' 16'''' 36''$: : AZ^2 : AT^2 . And the sum of all the mean horary motions throughout the year : sum of as many $16'' 35'''$, &c. :: sum of all the AZ^2 : sum of as many AT^2 :: (by Lem. 1. pr. 39.) 1 : 2; for the diurnal rotation, and annual motion of the earth, are supposed uniform.

[ib. But because the plane of the equator.] For by prop. 30, the disturbing force is as $3PK$, (and in that prop. the inclination of the planes being small is not taken notice of,) but when the plane of the orbit of P is inclined to the ecliptic, the line PK is projected into pK , and the force will therefore be diminished, in the ratio of PK to pK , or as the radius to the cosine of inclination; for the difference of the distances of P and T from the sun become less in that proportion, and the difference of the forces are diminished in the same ratio.

A C O M M E N T O N

Fig. This being a proposition of great difficulty, depending upon very intricate calculations, which few people are judges of; for this reason, the Author's demonstrations have been objected to, and censured by some people as not true. And Mr. Simpson (in his Miscellaneous Tracts) mentioning some of these Cavillers, falls into the same notion. He has invented a sort of motion (which he calls *Momentum*) unknown to Sir I. Newton, or any body else, and which differs from Sir Isaac's, in the ratio of 800000 to 925725. But I never before heard of any motion that was not made up of the quantity of matter and velocity.

Another frivolous objection he makes, is about the motion of a ring being different from that of the equator. And he tells us, that the motion (*Momentum*) of a ring round its diameter is only half of what it would be, when revolving in its plane round the center. But it is more than half, for it is as 1 to $\frac{3.1416}{2}$, as is demonstrated in

Art. III. What he writes afterwards (tho' he says, *it is evident,*) is not intelligible. But he concludes at last in his way, that Sir I. Newton has made the precession (by the sun's force) to be but half of what it should be. And as some he mentions had made the whole proposition erroneous, he modestly ascribes but *two mistakes* to Sir Isaac in this one proposition. But I believe, that whoever reads the foregoing Notes, will soon be convinced, that his demonstrations are all right, and that all these blunders they tell us of, are entirely of their own making, and must be ascribed to themselves only.

[Lem. 4. p. 326. Wherefore if both the quantity of light,] for quantity of light is as the quantity the body receives from the sun directly, and the square of the distance from the body reciprocally;

B. III. THE PRINCIPIA. 101

cally; that is, \propto square of the apparent diameter Fig. directly, and square of the body's distance from us, reciprocally: therefore the body's distance is as the apparent diameter directly, and $\sqrt{\text{light reciprocal}}$.

[Pr. 40. cor. 4.] For then the parabola and earth's orbit touch in the vertex, and the area there will be $\frac{1}{2}$ velocity \times distance or radius or $\frac{1}{4}$ parameter, that is $= \frac{10000 \times 243.2747}{2}$, and

$\frac{10000 \times 10.1364^{\frac{1}{2}}}{2}$ for the diurnal and horary motion. The rest is plain from Pr. 14. L. I.

[Lem. 5.] For the ordinates AH, BI, CK, &c. Put $a, 2a, 3a, 4a, 5a, 6a$. and take the differences as follows.

$$\begin{array}{ccccccccc} a & 2a & 3a & 4a & 5a & 6a \\ b & 2b & 3b & 4b & 5b & \\ c & 2c & 3c & 4c & & \\ d & 2d & 3d & & & \\ e & 2e & & & & \\ f. & & & & & \end{array}$$

Now, to find any ordinate as $4a$, we have $4a = 3a - 3b = 2a - 2b = a - b$
 $- 2b + 2c - 2 \times b + 2 \times c$

$+ d = a - 3.b + 3.c - d$.
 and so of others.

69.

A C O M M E N T . O N

Fig. Case I. If we substitute for $p, q, r, \&c.$ their
69. equals, we shall have

$$\begin{aligned} RS &= a \\ &+ b x - SH \\ &+ c x - SH x - \frac{IS}{2} \\ &+ d x - SH x - \frac{SI}{2} \times \frac{SK}{3} \\ &+ e x - SH x - \frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \\ &+ f x - SH x - \frac{SI}{2} \times \frac{SK}{3} \times \frac{SL}{4} \times \frac{SM}{5} \&c. \end{aligned}$$

Now let RS fall upon any ordinate, as for example on LD, then SH, SI, SK, SL, &c. will be $= LH, LI, LK, o, \&c. = 3, 2, 1, o, \&c.$ respectively, which being written in the aforesaid series, we have

$$RS = a - 3.b - 3 \times \frac{-2}{2}c - 3 \times \frac{-2}{2} \times \frac{-1}{3}d$$

$+ oe = a - 3.b + 3.c - d$, which, by what went before, is the value of the ordinate DL ; and since this holds generally, it is plain, this series will give the value of RS wherever it falls.

Case II. Let x represent any base HS, y any ordinate RS. Assume $y = A + Bx + Cx \times x - P + Dx \times x - P, x - Q$, continued to as many terms as there are ordinates, which suppose four; P, Q, R, &c. being respectively $= HI, HK, HL$. Then putting AH, BI, CK, DL, successively for y , and o, HI, HK, HL for x ; we shall have from the general equation, AH $= A, BI = A + BP, CK = A + BQ + CQ \times Q - P, DL = A + BR + CR \times R - P + DR \times R - P \times R - Q$, that is, from the figure,

AH

$$AH \doteq A = a,$$

$$BI = A + B \times HI.$$

$$CK = A + B \times HK + C \times HK \times IK.$$

$$DL = A + B \times HL + C \times HL \times IL + D \times HL \times IL \times KL.$$

Whence by Subtraction.

$$AH - BI = - B \times HI,$$

$$BI - CK = - B \times IK - C \times HK \times IK.$$

$$CK - DL = - B \times KL - C \times \overline{HK + IL} \times KL \\ - D \times HL \times IL \times KL.$$

Then dividing by the coefficients of B.

$$\frac{AH - BI}{HI} = - B = b,$$

$$\frac{BI - CK}{IK} = - B - C \times HK = 2b,$$

$$\frac{CK - DL}{KL} = - B - C \times \overline{HK + IL} - D \times HI \\ \times IL = 3b.$$

Again by Subtraction.

$$b - 2b = C \times HK.$$

$2b - 3b = C \times IL + D \times HL \times IL.$ And dividing

$$\frac{b - 2b}{HK} = C = c.$$

$$\frac{2b - 3b}{IL} = C + D \times HL = 2c. \text{ Also}$$

$$c - 2c = - D \times HL, \text{ and}$$

$$\frac{c - 2c}{HL} = - D = d.$$

Thus the coefficients A, B, C, &c. are determined for four ordinates, and they are found the same way, if more ordinates are given. Then taking x at pleasure, as suppose = HS, then RS or y will be $a + bx + cx \times x - P + dx \times x - P$

$\text{Fig. } \times x - Q, \&c. = a + b \times p + c \times p \times IS + d \times p \times IS \times SK, \&c. = a + bp + cq + dr, \&c.$

69. [Cor. Lem. 8.] for then $AC : A\delta (= AI - \delta I) :: A\mu C : A\delta dy - d\mu x (= AI\mu y - \delta I\mu X)$, and :: $ASC : AS\delta$ (Propor. 10.) :: $(A\mu C + ASC =) ASC\mu A : (A\delta dy - d\mu x + AS\delta =) AS\delta dy A - d\mu x =$ (because Δ 's $x\delta\mu$ and $\delta S\mu$ are equal) $AS\delta dy A - d\mu x + x\delta\mu - S\delta\mu = AS\delta dy A - S\delta\mu =) AS\mu dy A : \text{time of describing } A\mu C : \text{time of describing } A\mu y.$

[ib. Schol.] For Bn will cut the chord AC in a point a little beyond E (towards C), suppose at e : Now the area $AEX\mu A$ to the area $ACY\mu A$, is in something a greater ratio than AE to AC , more near than before. Therefore $ASEX\mu A : ASCY\mu A :: Ae : AC$. Whence $ASBX\mu A$ (or $ASEX\mu A$) : $ASCY\mu A :: Ae : AC$. Or $A^2BY\mu A : ASCY\mu A :: (Ae : AC ::) \text{ time in } AB : \text{time in } AC$, very near.

[Lem. 10. to the triangle ASC , that is,] $AC\mu A : ASC :: AC \times \frac{2}{3}M\mu : AC \times \frac{1}{2}SM$ (because the \angle 's SMA and $AI\mu$ are equal, and $M\mu = I\mu$) :: $AC \times \frac{1}{3}M\mu : AC \times SM :: MN : SM$. And by composition $ASC\mu A : ASC :: SN : SM$.

[Lem. 11. Subduplicate proportion of 1 to 2,] and therefore $= \frac{AC}{\sqrt{2}}$. And the arch in half the time $= \frac{AI}{\sqrt{2}}$. And $\frac{AI^2}{2} \div 2SP = \frac{AI^2}{4SP} = \text{space described in descending.}$

[Prop 41. and BE, by Lem. 11, is a portion,] tS and this hypotenuse (mentioned in the prop.) are the distances of the earth and comet from the sun. Now BE to tV is compounded of BE to the part of the hypotenuse (projected into BE) or of BS to the whole hypotenuse, and of that part of the hypotenuse to tV , that is, as the gravitating forces

forces at the comet and earth (for these lines would be described by falling bodies in equal times, by these forces), that is, as St^2 to hypotenuse square. And therefore $BE : tV :: BS \times St^2 : \text{hypotenuse cube}$.

Further, because of the immense distance of S , 70 any of the points B, i, μ , are nearly in the curve of the parabola, and μ the vertex, and $\mu p, i\lambda, q\sigma$ nearly parallel: Therefore $\angle I\mu i =$ (by reason of the rectangle $Ii\mu\lambda$) $< p\mu L$ or $= S\mu L$. Therefore μl is the diameter to the vertex μ ; also $\xi\sigma =$ (by construction) $3S\sigma + 3i\lambda = 3S\sigma + 3I\mu = 3S\sigma + 3\mu p =$ (because $2O\sigma = 3\mu p$) $3S\sigma + 2O\sigma$. And therefore $\xi O = 3S\sigma + 3O\sigma = 3OS$. But $\mu O : I\mu : : p\sigma : Ip$. But μl is bisected by $i\lambda$; therefore $Ip = 2I\lambda$; and $p\sigma = I\lambda = \frac{1}{2}Ip$ (since $I\sigma = 3l\lambda$ by construction); therefore $\mu O = \frac{1}{2}\mu l$. Whence (by Lem. 8.) a line drawn from ξ divides the chord AC nearly as the time; whence BE drawn towards ξ is rightly drawn. But (by construction) new $BE :$ former $BE :: BS^2 : S\mu + \frac{1}{3}i\lambda^2 ::$ as the gravitating force at the distance $S\mu + \frac{1}{3}i\lambda$: to gravitating force at the distance B or μ : : space fallen thro' at the distance $S\mu + \frac{1}{3}i\lambda$, in the time of describing the arch $\frac{1}{2}A\mu C$: space fallen thro' at B or μ , in the same time. But former $BE =$ space fallen thro' at μ . Therefore new $BE =$ space fallen thro' at the distance $S\mu + \frac{1}{3}i\lambda$, (in half the time the comet describes the arch intercepted between $TA, \tau C$ = (by Lem. 11.) $i\lambda$, or this new BE nearly; which therefore is rightly determined, nearer than before.

Because the distance of the sun and comet is something more than the hypotenuse of the triangle, whose base is $S\mu$ and perpendicular IO (because O is only a point in the chord, which should be in the arch), therefore he supposes that distance = hypotenuse (whose base is $S\mu + \frac{2}{3}i\lambda$, and perpendicular

Fig. perpendicular IO , that is) $= DO$. Also $MP : X ::$ (by 70. construction) $\sqrt{St} : \sqrt{OD} ::$ (by cor. 6. pr. 16. I.) velocity of the comet at distance DO : velocity thereof at distance St : : space described (uniformly) at the distance DO , and in the time of describing the arch ABC : space described (uniformly) at the distance St , in the same time, which is $= X$ by construction ; and therefore $MP =$ space described (uniformly) at the distance DO , (in the time that the arch ABC is described) $=$ (by Lem. 10. cor.) to the chord of that arch.

The points e, a, c, g ; and $\epsilon, \alpha, \chi, \gamma$, being found out as E, A, C, G , before; is with an intent to find at last $MP = MN$, or $AC = AG$. Wherefore several of them being thus found, and a simple curve (viz. a circle) drawn thro' these points, finds the points Z and X where those lines would be equal; for the nearer they approximate to equality, the nearer they come to the true points of the comet's orbit.

Further, velocity of a comet at Q (in a parabola) : velocity of a comet at Q (in a circle) by cor. 7. pr. 16. I.) :: $\sqrt{2} : 1 ::$ (nearly as) $\frac{4}{3} : 1$. Also velocity at Q (in a circle) : velocity at t (in a circle, by cor. 6. pr. 4. I.) :: $\sqrt{St} : \sqrt{SQ}$. Therefore, *ex equo*, velocity of the comet at Q , in a parabola : velocity of the earth in its orbit at t :: $\frac{4}{3}\sqrt{St} : \sqrt{SQ}$. But velocity of the comet at Q : velocity of the earth at t :: (nearly as) $AC : T\tau$. Therefore $AC : T\tau :: \frac{4}{3}\sqrt{St} : \sqrt{SQ}$; or $AC : \frac{4}{3}T\tau :: \sqrt{St} : \sqrt{SQ}$. As it is by construction (for AC is to $\frac{4}{3}T\tau$ in the reciprocal subduplicate ratio of SQ to St , and not in the direct subduplicate ratio, as is falsely printed; therefore Q is (nearly) in the chord of the parabola, and B a point of the comet's orbit, nearly).

Lastly, if $MP = MN$, or $AG = AC$. Then $Yb : YB :: Ye : YE :: ac : AC ::$ velocity in b : velo-

B. III. THE PRINCIPIA. 107

velocity in B : : $\sqrt{SB} : \sqrt{Sb}$. But if $Sb = SB$ Fig. and MP or AG invariable; it will be $Yb : YB ::$ as 70.

as or AG : AC, when the point G falls in CY.

Therefore universally $Yb : YB :: AG \times \sqrt{SB} : AC \times \sqrt{Sb} :: MP \times \sqrt{SB} : MN \times \sqrt{Sb}$, to find the point b true.

[ib. p. 352. may be seen in the following table.]

The places of the comet in this and all the other tables, are the geocentric longitudes and latitudes of the comet.

[ib. p. 358. And thinking, &c.] In plate 18, while the comet passes thro' I, K, L, M, &c. the earth passes from between P and K towards G. Whence the comet will move swift thro' LKL, and NOP; flower thro' LBM, and QRS, as is said in the foregoing page 357.

[ib. p. 360, as about 6 to 1000,] for by page 348, 9. $\frac{\text{lat. rect.}}{4} = 6$, nearly; when earth's dist.

$= 1000$.

[ib. p. 366. I found, that at the height, &c.] 7¹³

Let SA = rad. earth = 20950000 English feet, nearly, AB = 850, AF = AS, and the densities AH and BI are as 33 to 32. But (by cor. 2. p.

22. II.) $Aa - Bb : Aa - Ff :: tbin : tbnz$; that is, $AB \times SF$, or $2AB \times SA : AF \times SB$, or $SB \times SA :: tbin : tbnz$; or $2AB : SB :: tbin : tbnz$; that is, $1 : 12324 :: tbin : tbnz ::$ (by 1chol. prop. 86. hyper-

bola,) $\log. St - \log. Su : \log. St - \log. Sz :: 013364 : 1.518514 - \log. Sz$. Whence $\log. Sz =$

$- 164.820578$; and SZ or FN =, (0^{163} cyphers) 66 +. Wherefore FN : AH :: $0^{163} 66 : 33 ::$ or as 2 : $10^{165} :: 1 : 10^{164} ::$ density in F : den-

sity in A. But diameter of Saturn's orb is = 191 times the semidiameter of the orb's magnus = 191×20000 radii of the earth = $191 \times 20000 \times 20950000 \times 12$ inches = $960'' +$ inches. And a sphere of 1 inch diameter : to this sphere :: as 1 :

Fig. $\frac{960^{13}}{13} :: 1 : 880^{43} +$; but $10^{164} > 80^{44}$. Therefore, &c.

[Prop. 42.] the correction of the orbit here given is computed by the rule of false; 1st, for the longitude of the node, by operation 1 and 2, and 2dly for the inclination of the orbit, by operation 1 and 3.

I. for the longitude of the node; since (by construction) time of describing D : time of describing E :: D : E :: G : 1; and (time of describing E, nearly =) time B : time A :: 1 : C; therefore (*ex quo*) time of describing D : time A :: G : C; therefore G — C is the error of the times between the first and second observations, for the given longitude of the node K, and the given inclination of the orbit I. Again,

Time of describing d : time of describing e :: d : e :: g : 1; and time of describing e (nearly =) time B : A :: 1 : C; therefore time of describing d : time A :: g : C; and g — C is the error of time, between the two first observations, for the longitude K + P, and inclination I; therefore by the rule of false, by two positions (which says, diff. er. : diff. positions :: either er. : correction of its position), $G - g : G - C :: P : \frac{G - C}{G - g} P =$ the

correction of K, or the computation may be thus, since to the supposition K and K + P, the errors between the first and third observations, in these two cases, are T — S, and t — S; therefore $T - t : T - S :: P : \frac{T - S}{T - t} P =$ the same

correction of K; therefore mP (is to be) $= \frac{T - S}{T - t} P = \frac{G - C}{G - g} P$; and therefore $mT - mt = T - S$; and $mG - mg = G - C$.

II. For

II. For the inclination of the orbit, it is plain Fig. $T - S =$ first error of the times between the first and third observations, to the given node K, and inclination I; and $r - S =$ error between the first and third observation, to the node K, and inclination $I + Q$. Therefore (by the rule of false),

$$T - \tau : T - S :: Q : \frac{T - S}{T - \tau} Q = \text{correction of}$$

I; again this time of describing $\delta : \text{time of describing } \epsilon :: \delta : \epsilon :: \gamma : 1$; and (time of describing $\epsilon =$ nearly) time B : A :: 1 : C: ergo time of describing $\delta : A :: \gamma : C$; therefore $\gamma - C =$ error of the times between the first and second observations, to the node K and inclination $I + Q$; and $G - C$ was the error between the first and second observations, to the node K and inclination I; therefore (as before) $G - \gamma : G - C :: Q : \frac{G - C}{G - \gamma} Q =$ correction of I; therefore nQ (must be) $= \frac{T - S}{T - \tau} Q$

$$= \frac{G - C}{G - \gamma} Q; \text{ and } nT - n\tau = T - S, \text{ and } nG - ny = G - C. \text{ Lastly, from I and II.) } \\ 2T - 2S = mT - mt + nT - n\tau; \text{ and } 2G - 2C = mG - mg + nG - ny; \text{ therefore the numbers } m, n \text{ are rightly found.}$$

[ib. and lastly, if in—] because the longitudes in the first and second orbits are K and $K + mP$, therefore the latus rectums will be R and $R + r - R \times m$; and because in the first and third orbits, or planes thereof, the inclinations are I and $I + nQ$; therefore the latus rectums will be R and $R + \rho - R \times n$; or $R + r - R \times m$, and $R + r - R \times m + \rho - R \times n$.

Or thus; if the longitude K be increased (in op. 1, 2.) by P, then (because the distance of the per-

perihelion from the sun is increased accordingly), the latus rectum will be increased by $r - R$, and if the longitude be increased by mP , the latus rectum will be increased by $mr - R$. And for the same reason (in op. 1, 3.) if the inclination be increased to nQ , the latus rectum will be increased to $N_p - R$ (for at the increase Q , the increase of the latus rectum is $=_p - R$, since the whole is ε , or $R + \varepsilon - R$); therefore if both longitude and inclination be increased, the latus rectum will be increased in both these respects, and will become $R + mr - mR + n_p - nR$.

Also when K becomes $K + P$, the transverse becomes $\frac{I}{L + I - L}$ or $\frac{I}{L}$; also when K becomes $K + mP$ the transverse becomes $\frac{I}{L + m.l - L}$, for the same reason when I is increased to $I + nQ$, the transverse is $\frac{I}{L + n.\lambda - L}$. Therefore when both longitude and inclination are increased, the transverse will then be $\frac{I}{L + ml - mL + n\lambda - nL}$.

[ib. Sch. pa. 385, the comet] for by pa. 360, comet's dist. : earth dist. from $\odot :: 6 : 1000 ::$ dist. comet in \odot diameters : (earth's dist. in \odot diam. =) 109 \odot diam. therefore comet's dist. from the sun's center = $\frac{654}{1000} = 654$ diams. and from its surface

$$= 154 \text{ diams.} = \frac{1}{6} \text{ sun's diameter.}$$

A D E F E N C E o f
Sir I. N E W T O N,

A G A I N S T T H E

O B J E C T I O N S that have been made against
several Parts of the P R I N C I P I A.

*Raro antecedentem scelestum,
Deseruit pede pæna clando.* HOR. Lib. III.

T H I S incomparable Treatise being written in a concise stile, and in the synthetic method, and being upon subjects quite new and untouched before ; the generality of readers could make little of it. As it contained a new system of Philosophy, built upon the most sublime Geometry, the greatest mathematicians were obliged to study it with great care and attention ; and few became masters of the subject ; so that for a long time it was little read. But at last, when the value of it became more known, it gained universal approbation ; and the whole world stood amazed at the numberless new discoveries contained therein. And upon account of its universal agreement with all the phænomena of nature ; it was adopted as the only true system by all, except some few that, thro' envy or ignorance, were bigotted to some other scheme.

It is true, many things therein depend upon very difficult mathematical computations, not easily comprehended by every reader ; and this has given occasion to some people to question or even deny the truth of several propositions laid down in this book ;

book ; and have therefore given computations of their own, different from his, which they assert for truth ; and thereby, as they think, proving his to be false. I shall therefore spend a little time in answering the most material of these objections, and shall shew that these principles objected against, are all true in the sense Sir Isaac meant them ; and that these objections are only the effect of their own ignorance or inattention.

J. Bernoulli (Tom. IV. pa. 340) thinks Newton's demonstration, of cor. 4th to the laws, is imperfect, where he says, *only the action of two bodies is considered.* And lays down a method, by which he thinks it may be fully demonstrated. But at last leaves it undemonstrated himself ; and tells you, that this is the foundation, but leaves the calculation for others to make out.

But Sir I. Newton, after he had shewn the truth of the cor. in two bodies, shews that the common center of gravity of these two bodies and a third, will either be at rest, or move uniformly in a right line. And from thence, that the center of gravity of *any two* (in a system of bodies) suffers no change of state by their actions upon one another. From whence it follows, that in the whole system, all these actions (composed of the action of *every two*) can induce no change. And if the actions of *any two* makes no difference in the center of gravity of the whole, separately considered ; then the actions of *every two* considered collectively can make no difference. This is certainly so, tho' delivered in a few words, Sir Isaac was not writing to children.

The same Bernoulli (pa. 347) objects against the 10th prop. B. II. for not being solved in a way general enough to please him. Here Sir Isaac has solved the problem according to that law of resistance which really exists ; but could not spare time

time, or fill up his book with useless enquiries, or things of no consequence. His noble structure the *Principia* is all gold, but he has left the dross for such authors as this, who are fond of any thing, and will serve well enough for building up their systems. This author in particular is always depreciating Sir Isaac, and extolling his own performances; tho' they are long, tedious and laborious to the last degree, and often false; altho' he has the advantage of the analytical and fluxional methods; and seems to be ignorant that the method of analysis, by which he (*Bernoulli*) solves these problems, is incomparably shorter than the method of composition, in which Sir Isaac has writ. Had he written the Principia in the analitic way, or according to the method of Fluxions, a science then utterly unknown to the world; it could not have been read by any man living. By the same way of reasoning this boasting mathematical bully may as well condemn *Euclid's Elements*, or *Apollonius' Conics*; because most of the propositions may be demonstrated incomparably shorter by common Algebra.

This Author seems to have a particular spite against the English mathematicians, being always carping and criticising. His works are full of invectives against Sir I. Newton for his great discoveries, which this low Critic is unsuccessfully endeavouring to imitate. When one person has another man's works to look thro', he must be a great blunderbus that cannot make some small additions; a man placed upon another's shoulders, will see further than his supporter. Yet in many cases he has disimproved Sir I. Newton. Sir Isaac had a work entirely new to execute, in a short, and general way; and could not take notice of every bauble. This author has had nothing to do but to imitate. The regular works of the one will live

thro' all ages ; but the confused chaos of the other will sink into everlasting oblivion.

The same author (pa. 484) also objects against the demonstration of the 36th prop. B. II. For says he, *since in Newton's cataract there is no compression of the water in any place, nor against the sides of the cataract, the external water pressing inwards, must disturb the cataract, and mix with it.* And therefore the Newtonian explication being contrary to the laws of Hydrostatics cannot subsist. Yet what Newton says (Case I.) might have satisfied him. His words are ; *For let the ice in the vessel dissolve into water, yet will the efflux of the water, as to its velocity, remain the same as before. It will not be less, because the ice, now dissolved, will endeavour to descend. It will not be greater, because the ice now become water, cannot descend without hindering the descent of other water, equal to its own descent.* But this author is eternally cavilling at every thing ; and all his whole section on this subject, consisting of about 100 pages, where he has tost this prob. about into all forms, is nothing but a heap of absurd inconsistent stuff, neither agreeable to theory nor experiment. So we shall leave him here drowned in a gurge of his own contriving.

In the demonstration of prop. 47, B. II. Sir I. Newton assumes for a principle, that the parts of the air hath a motion excited in it, by some cause or other, according to the laws of an oscillating pendulum. But Euler has found fault with him for making such an hypothesis ; and has himself assumed some different hypotheses, from which he has pretended to demonstrate, that the motion of the particles of air, are moved according to such hypotheses ; and that his demonstration for his hypothesis, is as valid as Newton's is for his. But Sir I. Newton's hypothesis is more than an assumed one, for it is true in fact. For any tremulous or vibrating

vibrating body, from which sound is propagated, is known to vibrate according to this law, assumed by Newton ; and the like motion is communicated to the particles of air. Therefore he has rightly assumed, that the medium has such a motion excited in it ; and consequently that it will continue to move according to that law. But there is no such law of a vibrating body as *Euler* assumes, and therefore his demonstrations come to nothing. To find the velocity of sound is a very subtle problem ; and they that would see it truly resolved, may find it in my book of Fluxions, prob. 23, Sect. III.

In B. II. prop. 52, Sir I. Newton has investigated the properties of vortices, in order to see whether the celestial motions can be accounted for thereby. And he has shewn that the celestial bodies are not carried round in vortices. But *I. Bernoulli*, a person of eternal contradiction, has undertaken to defend the notion of vortexes ; tho' shewn to be absurd and impossible. Let him frame what hypothesis he will for the planets, he must find vortexes meeting one another for the comets, that move all manner of ways. Therefore to clear this point, he must have his vortexes made of such fluid matter, as has neither friction nor resistance, and that one part be penetrable to another part : that is, such a fluid as cannot be found in the world. Such is the blinded, bigotted, prejudice of some people, that the clearest demonstrations cannot cure their madness. If any body asks one of this stamp, how the planets move round, they presently answer, in vortexes. But ask them what causes the vortexes to move round, here they are at a stand, and have nothing to say, but that they move by their own nature. But they might as well have said at first that the planets move round by their own nature, if they can give no better

account of them, and so save the labour of constructing these useless vortices.

Another author that is *vortex-mad*, is *Euler*, and he seems to go beyond any of the rest, for he cannot account for the rising of the tides without vortexes. But he has not shewn us by what extraordinary mechanism, or invisible wheelwork his vortices are constituted, so as to be able to produce the tides, or cause the motion of the comets, since these vortices must all run counter to one another, and penetrate one another, and yet miraculously preserve their motions entire.

This idle notion was first introduced in the time of ignorance, upon the supposition of a *fuga vacui*, or that nature abhorred a *vacuum*. But by our better acquaintance with the nature and properties of body, and the laws of motion; we now know, that the operations of nature cannot be performed in a *plenum*; and therefore a *vacuum* is absolutely necessary.

If we had known of no celestial bodies but the planets moving all one way, the supposition of a vortex to carry them about, clumsy as it is, might have passed for possible. But one would have thought, that the comet's moving all manner of ways, would have cured this delirious notion; and have taught them the impossibility of such a scheme. But these authors, these defenders of vortices, are so hardy, that they are not at all afraid of an absurdity or a contradiction. These things do not affect or touch them in the least; but they go on unconcerned in their usual way, tho' contrary to all the laws of nature.

Did I say that the theory of vortices had its original from the principle of a *fuga vacui*? I did. And the principle of a *fuga vacui* had its rise from the phenomena of water rising in pumps and siphons, as in the *Torricellian experiment*. Here for want of trying such experiments to their full extent, the principle of

of a *fuga vacui* was assumed by these philosophers, as the genuine cause of these effects. So that, upon no better a footing than this, tho' the true cause of these effects (the external pressure of the air) be now perfectly known, yet with these bigotted people, the same false principle still exists.

These philosophers (if they can be called such) are endeavouring, contrary to the nature of things, to find out the most complicate causes for explaining the phenomena, instead of seeking the simplest causes; and they seem utterly to reject all simple causes, which are the greatest beauty of nature. For if their brains were not turned round in a vortex, they could never prefer these complex vortical schemes before the simple doctrine of projectile and centripetal forces. Such Philosophers!

Sir I. Newton had shewn in the schol. of Prop. 14. B. III. that the aphelions of the interior planets move a little *in consequentia*, (by the actions of Jupiter and Saturn) in the sesquiplicate ratio of their distances from the sun. But *Bernoulli* says this is false, for it holds not true in Saturn. And yet Sir Isaac tells him expressly, it is the inferior planets that observe that law. So little do some men care what they write, if they can only put on an air of contradiction.

In finding the proportion of the axis and diameter of the earth, Prop. 19. B. III. Sir Isaac assumes them to be as 100 to 101, and from thence finds the excess of weight, in one above the other, to be $\frac{4}{505}$ parts. Therefore he makes this propor-

tion $\frac{4}{505} : \frac{1}{100} :: \text{the force } \frac{1}{289} : \frac{1}{229}$ the true excess of height. Yet some people have objected against this as a wrong way. But herein they are much mistaken; for if any force be increased, the effect will be increased in the same ratio; and there-

fore increasing the centrifugal force here, will increase the difference of the heights, in the very same proportion; as these quantities are very small. Upon this account, some people have unnecessarily run into long calculations, and only came at last to the same conclusion.

And the same argument holds good against such as have objected against finding the height of the tide (cor. Prop. 36. B. III.) by the rule of proportion, comparing it with the centrifugal force. For all forces of whatever kind will produce effects proportional to the *quantities* of these forces; and therefore he has rightly found the height of the tide, by that proportion.

But Sir I. Newton's explanation of the tides (Prop. 24. B. III.) does not please Euler, tho' he accounts for every circumstance thereof. He thinks ascribing these effects to the actions of the sun and moon, is recurring to *occult causes*, and therefore he had rather recur to *vortexes* for the explanation thereof; the notion of which has been confuted over and over. He denies the gravitation of bodies towards one another, because he cannot discover the cause of gravity; and therefore he will not allow it to have any thing to do with the matter, as being an occult quality. But he recurs to a principle that is more than occult, his incomprehensible vortices; which he thinks the tides are raised by; tho' he has not attempted to explain in what manner his vortices can do it.

He says, Sir I. Newton's account of the tides is not sufficiently explained, that any certain judgment can be formed whether it is true or false; that he did not explain the phenomena of the tides but only darkened them. But certainly this man does not *know* what he writes, or does not *care* what he writes. Sir Isaac explained all the principal phenomena thereof, and has shewn, that his Theory

Theory is agreeable to observations both in nature and quantity. And all matters of less note may be easily deduced from his general Theory, by any intelligent person. But Sir Isaac did not choose to erect any *pontes a finorum*.

This author values himself very much for rejecting the principle of *attraction*, established by *some Englishmen*, because it is an occult quality. This profound Philosopher will either know every thing or nothing; he will make no use of the forces of gravity, because he knows not the cause of gravity. For the very same reason he would want to know the cause of that cause; and so he must know every thing in the whole chain of causes and effects, or he cannot be satisfied. This odd temper in some men arises from the pride of the human mind, which attempts to soar above its sphere. They disdain to know the little matter that is within their power to know; whilst they are continually aspiring at things that are without the reach of their knowledge, things for which they have no *faculties* suited to understand, and no *data* to determine. This is not to philosophise, but to trifle.

As this Gentleman takes the liberty to sneer at the *English*, he may please to stay and take this observation along with him, that if it had not been for an *Englishman*, he (and such like) had known nothing. To him they are beholden for all these great and wonderful discoveries, which all the world acknowledge and praise him for, except these sons of detraction, envy, and ingratitude. Discoveries that may be looked upon as the works of a Genius rather divine than human.

The *English* have found by experience, that by the power of some cause or other, all bodies are drawn or impelled towards one another, according to certain laws. And these laws are found out by observations, and a just reasoning from the laws of

motion. This cause, be what it will, they call *gravity*, without pretending to determine of what kind it is. If this author knew the physical cause of gravity never so well, he would be no wiser; for the effects of it would be just the same; and we can measure the effects of gravity, and from thence find the quantities, and proportions of the generating forces, without knowing what these forces consist in. So that the knowledge of such a cause would be only a useless speculation. But this author is so fearful of occult causes, that he dare not make use of their manifest effects; and condemns the English at once for believing their own eyes, that there are such effects flowing from this unknown cause, which they have agreed to call *gravity*. For it appears to them, that the effects thereof are all that they have any thing to do with. Sir I. Newton tells him more than once, *that he does not take upon him to define the kind or manner of action, or the causes or physical reason thereof*; but this author cannot take it in.

Some persons, when they can find nothing else to say, have cavilled about the term *gravity*, and so their objections are dwindled to a dispute about words, and not things. I suppose every thing in nature, that we have occasion to make the least use of in our discourse and reasoning, be it known or unknown, ought to have a name given it, to distinguish it from other things, and to convey our meaning to others. And thus the *cause* of the acceleration of bodies towards one another is expressed by the word *gravity*, and the action itself is called *gravitation*. The word *attraction* is used in the very same sense; that is, not in a *physical* but *mathematical* sense, which regards only the *quantity* of the cause. That these terms are the most proper that could be chosen, will appear from hence; the word *attraction* is taken from its most manifest quality; for a body

dy moving towards another with an accelerated motion, has all the appearance possible of being drawn towards that other by some inherent virtue in the other. The word *gravity* has evermore expressed the tendency of a heavy body to the earth; and therefore by parity of reason, will as properly express the tendency of the moon towards the earth; or of the earth towards the sun, or of any body towards another body.

Suppose this philosopher, or any other of that sort, was asked to calculate the times, spaces, or velocities of falling bodies; which is the most simple case that can be proposed about gravity. Would not any body justly laugh at him, if he stood to demur about it, and refuse to calculate, till he knew whether their falling was caused by attraction, impulsion, or the rotation of a vortex? And would not he equally deserve to be laughed at, that should hesitate to calculate the motions of the moon, or of the earth and planets; or of the tides; upon the same account; when they are all acted upon by the same unknown cause of gravity?

But to return to the tides. This Gentleman (*Euler*) tells us, that Newton's method is erroneous, by which he found the sea to rise to the height of near two feet, by the sun's force only. And says, that Newton found out this enormous effect, by comparing the sun's force with the centrifugal force of the earth. This has been answered before; and certainly this Gentleman knows little about the nature of forces, if he does not allow that two equal forces, of however different kinds, will always have equal effects; and proportional forces, proportional effects, especially in their nascent state. For it is not the *kind*, but the *quantity* of force that is to be regarded. Therefore Newton rightly found the solar tide near two feet, and the lunar tide $8\frac{1}{2}$ feet, agreeable to experience. But to shew you what sort of a theory this Gentleman works by, he finds

finds the solar tide only half a foot, and the lunar tide $2\frac{1}{4}$ feet, in all not three feet; which all observations confute, and with it, his erroneous method of computation. I have met with no body yet, but what makes it at least three times as much.

He also tells us, that Newton found out the forces of the sun and moon by help of the tides, but he has not done it accurately. And yet Newton took in every circumstance that could any way affect it; as may be seen in Prop. 37. B. III.

Having had occasion to compare different kinds of forces with one another; I will venture to lay down this as a general Rule, that all forces whatever, whether attractive or impulsive, centripetal or centrifugal, or of what kind soever, if they be equal, they will produce equal effects. And therefore how idle must it be for these men to wrangle about the *kind*, when the *quantity* only is concerned in the effect, and can only be of any real use to us in our calculations. The enquiry after the *kind* and *modus* of action, is a physical, or rather a metaphysical speculation, the knowledge of which they can never come at.

It has likewise been objected by some persons, that the two examples of Newton, for finding the tides, are ill chosen. But however, he had no more to choose on, and by their near agreement, it shews they were well chosen. Euler tells you, that at *Havre de Grace*, the greatest and least tides are as 17 to 11; and therefore the sun's force to the moon's, will be as 17 — 11 to 17 + 11, or as 6 to 28; or as he makes it, as 7.13 to 28, which is about as 1 to 4, a proportion not very different from Newton's. *Dan. Bernoulli* says, that at *St. Malo's*, the greatest height to the least, is as 50 to 15, which makes the sun's force to the moon's, as 35 to 65, or as 7 to 13, not so much as 1 to 2; a conclusion utterly inconsistent with all other observations;

servations; which argues, that the observation has not been made with sufficient accuracy. However this is certain, that if any place can be improper for such an experiment, this place is; by reason of the very extraordinary tides. For here the tide being hurried up a long channel, growing continually straiter, it is forced up to an unusual height.

However, I cannot think that it signifies a great deal, where, or in what places, these experiments are made, provided the sea be deep, and have free access and recess to and from the place of observation. For tho' the tides be higher in one place than another, the sun and moon conspire alike to that; for if the water be accelerated in any degree by the moon's force, it will likewise be accelerated proportionably by the sun's force; so that the result will be nearly the same.

I cannot but be surprised, that in so material a point as this, no body has been sent purposely, to proper places to make observations of the tides. Since by this method only, the forces of the sun and moon can be determined to any tolerable degree of exactness. No celestial observations can assist us in this matter. Astronomy affords us no help. And there seems to be no way for us, to gain this great point, but this method by the tides. The forces of the sun and moon are so very small, in respect to the force of gravity, that no common hydrostatical experiment can shew us the least effects thereof. It is only in the tides that their effects become sensible.

The forces of the sun and moon reach to the very center of the earth, and act upon every point of the radius or column of water under them; and diminish the gravity of every particle thereof; and all these forces conspire together to raise the tide; and therefore the radius of the earth becomes the proper scale for all these forces to act on. And

we

we cannot see the total effect of all these forces, unless we have a depth equal to the whole radius of the earth, or an extent of sea of 90 degrees; and this effect or the tides, when at the greatest, will then only amount to a few feet.

If I was to give directions for making observations of the tides for this purpose, I would advise to choose some place near the equinoctial, as on the coast of *Africa* or *America*. And the place would be most advantageous, where the sun and moon are in the zenith at the first observation; and in the horizon, at the second, for the spring tides. And one in the zenith and the other in the horizon, for two observations at the neap tides. And such places should be chosen where the sea is of large extent, and deep; so as to communicate freely with the place, back and forward. And such places will be best in a calm country, and performed in calm weather; and all circumstances of weather should be alike (as near as can be) at any two correspondent times of observation, the syzygies and quadratures. These observations should be made at the syzygies and quadratures, both day and night. And also when the tides are highest and lowest, which is three days after. But for some purposes, trial should be made every day. And the business of trying at the syzygies and quadratures, should be continued for a twelvemonth. And more places than one should be tried. Perhaps some islands in the middle of the sea, as *St. Helena*, may be proper; for the weather is likely to be more uniform, than on the continent. Having thus gained a sufficient number of observations, the best may be selected, and a mean ratio found, by which this matter will be finally determined.

There are some people that object against this method of finding the sun and moon's forces, by the tides; and reckon it very precarious, and subject

ject to many obstacles and intervening causes, by which the tides are perpetually influenced and disturbed; as if every thing had not its difficulties; the only disturbing cause is the wind. Yet they can tell us of no other method, but what is more precarious and more impracticable, and less exact. And so much for the tides.

The 39th prop. is about finding the precession of the equinoxes; against this, being a prob. of great difficulty, objections have been raised by several people, alledging that it is not truly demonstrated. Mr. *Simpson* is among this clan; and he absurdly makes the precession by the sun's force alone to be $21'' 6'''$, which is double to Sir I. Newton's; the consequence of which is, that the motion by the moon's force will be only $29''$. So that by this, the moon's force will be to the sun's, only as $1\frac{2}{3}$ to 1; yet he says in another place, that the moon's force to the sun's cannot be less than $2\frac{1}{2}$ to 1. So inconsistent and erroneous are his operations. But this has been taken notice of before in the Comment. But the moon's force must be greater than $2\frac{1}{2}$; for in all observations of the tides, that have been regularly made, the moon's force is 4 or more. And it is hardly possible in any observation of the tides, made with any tolerable degree of care, to miss near a half. They that would see the precession of the equinoxes truly calculated, will find it in Prob. 26, Sect. III. of my Fluxions.

Other objections of less moment I pass over, as their malignity and falsehood will appear to every reader. Such as the absurd opinion that motion cannot be lost in the world; (concerning which, see my Mechanics, 4to. prop. 10. and cors.) Also the false opinions of these that deny that the composition and resolution of forces, are analogous to the composition and resolution of motions, which are
their

their adequate effects. Their endless cavils against his method of demonstrating any proposition, which is the synthetical; preferring their own analytical methods as being shorter, which is no wonder. Their introducing into physical calculations, an obscure, precarious law or force, called *vis viva*; founded upon no certain principles. Concerning which see my Mechan. 4to. schol. to prop. 11, where the nature of it is unravelled. Their promiscuously using the words force and motion, for one another, which are as different as cause and effect; which induces no small obscurity into their writings. Their ascribing the invention of Fluxions to Leibnitz, contrary to the clearest testimonies; and some of them tell us, that, *not be that first found, but be that first published it, deserves the praise.* As if the publisher could have published it, at all, if the inventor had not first found it out. Hero the inventor is robbed of his due praise, to give it to the thief that stole it. And in general, their aggravating every trifling slip, as a capital crime; and instead of praising him for what he has done (which is more than all the world ever did before) they dispraise him for what he *has not* done, or had not time to do; and lampoon him because his time or his knowledge was *not infinite.*

But if any body should ask, what any of these *Bravo's* have done since? The answer is, nothing at all, or less than nothing; they have been turning science backwards. For they have been doing nothing but undermining his principles, (tho' built upon the surest foundations,) to introduce their own chimerical hypotheses, that have nothing to support them but impudence, ignorance and presumption.

At the end of the Principia, Sir I. Newton has given us his thoughts of the deity. Here he shews that God is an *eternal, infinite, and powerful being.*

being. That all the frame of nature is owing to him, which he made, and governs. And from the similitude of all the parts of the world, he shews that God is *one*. That he is a being acting with *council* and *design*, and with the greatest *wisdom*. That we have ideas only of his *attributes*, but know nothing of his *substance*; nor after what manner he acts or does any thing. Indeed we know nothing of the substance of any thing, much less of God. And as we cannot conceive of space and time, but as necessarily existing; much more must we allow that God exists necessarily; and consequently that he exists *always* and *every where*; and that not *partially* but *totally*. In short, he has laid down the best metaphysical notions of God, that can be met with any where.

But M. Leibnitz will not allow that God has in himself (much less other animals), any principle or power of acting, but as he is first acted on by some *motive*, which he thinks is to determine his actions. And in consequence of that, if two equal ways of acting were laid before God, he could choose neither; but be like a balance, acted on by two equal weights, that would turn no way. But at this rate God is not a *free agent*, but a mere *patient*. He reckons it an *imperfection* in God to be able to choose *one* out of two perfectly equal things; because he says there wants a *sufficient reason*; and therefore he can choose neither. But it is certainly a *greater imperfection* to choose *neither*, than to choose *one*. It is a principle with him that God must have a sufficient reason; which is true, but it is God's mere will that is the sufficient reason. And how comes he to know what is a sufficient reason to a perfectly free agent, that has it in his power to do any thing; as if man, who is no more than a *worm* in the creation, can presume to know what is fittest to be done.

Sir

Sir I. Newton has asserted, that all the great bodies in the world move in free space, which is unbounded and infinite. That all bodies have pores or empty spaces within them. But this author will not allow any *vacuum*, but will have the world be a perfect *plenum*. And he tells us also, that the world will continue for ever without any alteration (or mending); but he has taken particular care to prevent that, by introducing his *plenum*. The motions of the heavenly bodies must needs be retarded and soon stopt, in moving thro' dense matter, tho' never so fluid. So that one of his suppositions is inconsistent with another; and no body could more effectually destroy his own hypothesis, than he has done himself. He had had infinitely better chance for this supposition, if he had made the planets, &c. to move in a *vacuum*.

If God had designed every thing to stagnate, and be fixt in the world; then a *plenum* seems to be the best constitution for that end. But as all the operations of nature are to be performed by motion, which indeed is the beauty of nature; then placing them in *vacuo*, must needs be the best constitution, as the motions will then be free and unresisted, and the most durable possible. And therefore it is as necessary to have a *vacuum*, in which these motions are to be performed, as it is necessary to have *body* to perform these motions.

The same objections he has made against empty space, he makes against time; for with him space is no more than the order of co-existing beings; and time the order of successive ones; but at this rate the words *nearer* or *further*, *sooner* or *later*, signify nothing; yet both space and time are measured by quantity, and therefore are themselves quantities and consequently real beings, which confutes his notions. And indeed common sense is enough to determine this; for their existence is self-evident, even

even to the most ignorant; and cannot be made more evident by all the arguments in the world. Arguments in such cases, serve for nothing but perplexing things that are plain of themselves, and are often brought for that purpose. *Actions* and a *vacuum* are the first principles of the most ancient Philosophy; which is the same thing as to say, that God has made *bodies*, and *room* for them to act and to move in. For the whole world appears to our senses, to be nothing but *matter* and *motion*.

This learned Philosopher's argument against empty space is this; *every perfection which God could give to things, without derogating from their other perfections, he has given them.* Suppose then an empty space, God could have placed matter in it, (which is more excellent than empty space); therefore he has done it, and consequently there is no vacuum. Here he supposes that there is no excellency or perfection at all in motion; and so this world-maker will not agree to leave any room or free space for bodies to move in. And in this he is as dogmatical, as if he had been originally one of God Almighty's privy council. But had he gone one step further, he would have seen the absurdity of it. For God did not make the world to stand still; but to move after various laws and rules. And consequently when this acute Philosopher fills the planetary regions with matter, he *derogates from the perfections* of the other bodies, by destroying their motion, with his new matter. And in this case, and for this very purpose, empty space is more excellent than matter.

But the *ignis fatuus* that leads him astray, is the argument of *a sufficient reason*; when at the same time he is no judge at all what is sufficient. Yet this principle he brandishes about with no small ostentation and assurance. But it is of no more use in his hands, than the sword of *Achilles*, in the hands of an infant.

C O N C E R N I N G T H E
O P T I C S.

NO sooner did Sir *I. Newton* publish his theory of Light and Colours, than he was attacked on all sides. A doctrine so strange, and contrary to all the received opinions about the nature of light, could not be admitted. This brought him into many frivolous disputes, with injudicious people that had imbibed other principles. Yet he shewed the truth of his theory, by undeniable experiments, if his opposers had but had understanding enough to consider them. He had read lectures before, in the University of *Cambridge*, concerning light and colours, which he was thinking of publishing; but the disputes he was involved in, from what he had published in the Transactions, made him lay aside the thoughts of publishing any more. And so these lectures were laid by in the University, and were not printed till after his death. And in this I think he was much in the right; for who would ever spend any time or labour, to make men wise against their wills; and especially a set of men, that loved cavilling and disputing better than truth and knowledge.

By reason of these senseless disputes, he would not agree to print his book of Optics for many years; and at last it was only printed in English, with a design it should not go abroad, to create any fresh disputes; but was afterwards translated into Latin.

This

This book, tho' full of the most curious and profound discoveries in *Optics*, could not pass without the most severe scrutiny ; and I believe, hardly a proposition in the book but what was disputed by some rash Critic or other. But as the proofs were mostly experimental ; as soon as people had a mind to leave their prejudices, and try these experiments fairly and impartially ; they found them all true and agreeable to his theory, which at length began to gain ground ; and all disputes are now subsided.

As there are some things laid down in this book, without their demonstrations ; such of them as are difficult, are calculated in my book of Optics.

Towards the end of this book Sir Isaac tells us, the true method of philosophising, his thoughts of the original and constitution of the world, and of the Deity. He tells us the world could not arise out of a *Chaos*, by the mere laws of nature, but must be the work of a powerful, everliving agent, acting with wisdom and design. He tells us, that it appears from the phenomena, that *there is a being incorporeal, living, intelligent, omnipresent ; who in infinite space, as it were in his sensory, sees the things themselves intimately, by their immediate presence to him — whilst the images only of these things are carried thro' our organs of sense into our little sensoriums, and are there seen by that which in us perceives and thinks.*

But Mr. Leibnitz catching at the word *sensorium*, and perverting Sir Isaac's meaning, tells you, that Sir Isaac makes space to be the sensorium of God, as an organ, by which he perceives things, and thereby makes God the soul of the world, contrary to Sir Isaac's plain declaration ; whereas he speaks only by way of simile (*as it were, in his sensory*). For such is the imperfection of our notions concerning God, that when we would describe any

of his properties or attributes, we can only do it by similitude, comparing it with something similar to it in ourselves; which surely cannot be taken in a strict sense. Nothing is more common than to talk of ideas in the mind of God, when we speak of his knowledge or omniscience. Yet it is certain God has no need of ideas or images (as we have), when he is present to the things themselves. And I think it was hardly possible for Sir Isaac to find out a fitter simile, than to compare *infinite space* with the *place where God perceives all things*; just as the brain is the place where *men perceive the images (only) of things*. For suppose the brain to be the *organ of perception*, it is nevertheless the *place where these images are perceived*. This misconstruing the words of Sir I. Newton, shews the captiousness of the man; especially when he is told that *God is a uniform being, void of organs, members or parts, that all things in the world are his Creatures, subordinate to him, and subservient to his will. That he is no more the soul of them, than the soul of a man is the soul of the species of things, in the place of his sensation.* All this, one would think, should stop his mouth.

This Gentleman calls God a *supramundane intelligence*, which is only an unintelligible term to quack with. For certainly he exists *in the world*, but after what manner is a secret to us. If God have any existence, he exists *in space*; to say that he exists *out of space*, is to say that he does not exist at all, because space is every where. So does he exist *in time*; for to exist *out of time*, or in no time, is not to exist. He exists then in all space and in all time, that is *always and every where*.

Sir Isaac tells us blind fate could never make all the planets move one and the same way, in concentric orbs; some inconsiderable irregularities excepted, which may have risen from the mutual actions of comets and planets

nets upon one another, and which will be apt to increase, till this system wants a reformation.

Now these actions of the planets and comets on each other, and the consequent irregularities produced thereby, are matters of fact, and evident from observations. And it is as evident that they will increase, from the same causes. Yet Mr. Leibnitz is so ignorant of these effects, that he takes upon him not only to censure Sir I. Newton for making these observations, but God Almighty for not making his work otherwise; and compares him to an *imperfect workman*, that has his *watch* to wind up now and then. He thinks it an imperfection in God, if he does not make the world so as to last in the same state for ever. At this rate he may as well think it an imperfection in his Maker, if man does not last for ever, and live for eternity.

It is a bold assertion to say that God cannot make any thing to what degree of perfection he pleases, or to last as long as he pleases, or to reform, regulate, or alter it at his pleasure. Nothing will please this Gentleman but to make God to act *ad extremum*, as all necessary agents do, if they can be called agents. At this rate, not only the world must be made so perfect, as to last for eternity, and want no regulating, which is the thing he affirms; but even the bodies of men and other animals, for the same reason as was observed before, must last for ever, and never come to decay, or need any repairs; which is arguing against the clearest evidence of his senses; and is a demonstration of the absurdity of his hypothesis. So that in some cases it is true (what some men have objected against science in general), that the greatest philosophers are the greatest asses.

*Go, teach eternal wisdom how to rule,
Then drop into thyself, and be a fool.*

POPE.

A D E F E N C E O F

His Doctrine of *Indiscernibles* is altogether as wild a notion as the other of forcing God to do his best. His principle is this, that there are not in nature, two *real* absolute beings, *indiscernible* from each other, (that is, two such things as are perfectly alike). For if there were, he says, God and nature would act without a reason ; it is a thing contrary to the divine wisdom, and therefore they do not exist. And he instances in two leaves or two drops of water. And he seems to build his opinion upon observation, which is, that if you seek never so long, you cannot find two leaves alike ; and upon this he rashly concludes, that it is impossible ; because God would not be wise in doing of it. However, if he can give no wiser a reason than this, I doubt his principle will fall to the ground. For in the nature of things, what antipathy can God have against two equal or like things, any more than against two unequal or unlike things. It is impious thus to tie the hands of God, and vain in him to pretend to know what God's wisdom will choose or will not choose. His attempt to know this is as ridiculous as it would be, in attempting to move the whole world with the strength of a finger.

I believe the reader will hardly be satisfied, whilst I am endeavouring to confute his notion if I do not advance a better solution. This I shall attempt as follows, by a very simple and plain case. Suppose the length of the least sort of leaves to be a , of the greatest b ; their difference $b-a$ or d . Now I hope it will be allowed me, that the quantity or length d is infinitely divisible ; and consequently there is an infinite number of different lengths contained in the length d ; and hence there may be an infinite number of leaves, and all of different lengths, in a gradual increase from a to b . This being settled, let us consider the number

number of leaves. The number of them on any one tree is but finite, and the number of all the trees in the world is but finite; for the sands of the sea are numerable. Therefore all the leaves in the world make but a finite number. Suppose then any one length be named for a leaf; it will be the odds of infinity to 1, that in a geometrical exactness, there is not a leaf of that length in the world; because the number of *leaves* being short of the number of possible *cases*; there will be an infinite number of cases that do not happen; and therefore it is infinity to 1 against the happening of any one in particular. It follows likewise that, name the length of any one leaf, and it is the odds of infinity to 1, that another cannot be found *precisely* of the same length.

Now this calculation regards only the length, but there are other circumstances as variable, such as breadth, weight, colour, form, curvature, different arrangement of the veins, &c. and all diversified in quantities and ways infinitely various; therefore the odds will, by this means, be infinitely multiplied; so that it is next to impossible for such a thing to happen naturally. And here, we need not to have supposed infinite degrees of length; for if we had but assumed a very great number of lengths, each differing by the least conceivable quantity; yet by the infinite number of circumstances that attend it, the result would still shew that it is almost as infinity to 1, against such an event happening. And by the same reasoning one cannot find two stones alike, or two drops of water.

As to drops of water, there is not as great a variety as in leaves. Yet if you suppose but one heterogeneous particle; that single particle is capable of an infinite number of situations; and there may be 2, 3, &c. or any number of such particles;

from all which, the same conclusion will happen as before. And here we may observe, that the more compounded any body is, the greater is the improbability of this event happening.

Hence appears the true reason why such events do not happen, and that is, the infinite number of *possible* cases, and the finite number of *real* cases ; and upon the infinitely *small* chance there is for its happening ; but it is not at all from the impossibility of the thing ; which may be made plain by this instance. Suppose there be 100 dice, and any one should undertake to throw all the aces at once. The number of chances is so great against him, that in all probability, he would never do it while the world stands. Yet the thing is not impossible in itself ; for the dice may be so directed by art, as to be thrown every time.

So now the whole mystery is disclosed that seduced this learned Philosopher to abridge the *power* of God, for fear of contracting his *wisdom*. You may see that all the sizes or forms of leaves are alike indifferent to God, and nature has the same chance for any length or form. But what is all this to do with the *power* of God ? He can certainly make millions of things all equal and alike, if he has need of them in the creation, as easily as he can make them unequal or unlike. But the excuse this Gentleman has for this chimerical opinion, is this, he says *two things* perfectly alike will not be two things, but only *one thing*. But this is an assertion equally ridiculous, for two things will be as distinct, if they differ but in one property, as if they differed in ten thousand. Nothing can identify two things (*i. e.* reduce them to one), but existing in the very same place, at the very same time, and having all other properties the same.

Men are such imperfect artists, they cannot pretend

tend to make things equal or alike, to a geometrical exactness ; but they can make them so near as they cannot discover any difference. But God can work perfectly, which men can only imitate.

God is a being self-existent, omniscient, omnipotent, and omnipresent. This being can do all things that are possible to be done, or that imply no contradiction. These are his natural attributes, as has been mentioned before ; and which Sir I. Newton has given a sublime description of, both here and in the Principia, which this *babbler* has thought fit to criticise, but with an ill grace. As to his moral attributes, wherewith some people have invested and complimented God Almighty ; as his *Goodness*, *Mercy*, *Justice*, &c. these are not real properties, but are only consequent from, and included in, his wisdom. As for example, God cannot be said to be infinitely good, as he may be said to be infinitely powerful, or infinitely wise ; but he is always so far good, as is consistent with his wisdom ; and the like for his mercy, &c. As to our knowledge of God, we must content ourselves with the knowledge of his natural attributes ; for we know nothing further relating to him. It is a profound secret to us how he acts, how he perceives, or how he exists. Here he hides himself from us and will not be known. Nor should we even know that such a being exists at all, but from our own existence, and the regular phenomena that appear in the world, which bear witness to it. So that we are sure there must be a first cause, from whom and by whom all these things have their original ; as is shewn at large by our great author Sir I. Newton.

I believe the improvements that have been made to the *Science of Optics*, since he left it, are few or none ; his queries are still to seek, no body has investigated any of them. One would think that these

these cavillers against gravity, as an *occult quality*, would have taken care to have investigated its cause, hinted at in these queries; but it remains as occult as ever, for them. They are more ready to dispute than to act. To compleat these things, we should have another *Newton*; but as that is not to be expected, we need not think that any improvements will go fast forward.

As I have considered the principal objections that have been made against the *Principia* and the *Optics*, mostly by foreigners. It cannot be amiss if I take notice of some that have been made against his *Chronology*, by some in our own country; to shew how they *bite the file*.

*An Account of some of the numerous
Inconsistencies, contained in the Ob-
jections made by the Rev. Dr. Ruther-
forth, Regius Professor of Divinity
in the University of Cambridge, a-
gainst Sir I. Newton's Account of
the Argonautic Expedition.*

*Est genus hominum qui esse primos se omnium rerum
volunt, nec sunt :*

TERENT. Eun.

THE Professor sets off by telling us, that the whole of his (Sir Isaac's) argument, for settling the time of the Argonautic expedition, depends upon three principles. 1. That Chiron and Museus made a sphere. 2. That they drew the Colures of the equinoxes thro' the 15th degrees of Aries and Libra, and that of the Solstices thro' the 15th degrees of Cancer and Capricorn. 3. That they knew how to draw the Colures exactly.

Here he begins by asserting a manifest falsehood; for the whole of his argument does not consist in these three Articles. For Sir Isaac strengthens his argument by so many events, taken out of history, which all conspire to prove the same thing, that no body (except they be strangely bigotted, like this Gentleman) can refuse his assent. But let us examine his three grand principles. 1. That Chiron and Museus made a sphere. Now this is a point of history, and therefore ought to be taken as matter of

of fact, or else there is an end of all reasoning from historical relations. Yet our Doctor shews a strong inclination to suspect its truth, if it could be done; and seems to waver between two opinions, whether it was, or was not so. However, not being able to get over that, he tells us, that these figures drawn on the sphere, must relate to religion; *for delineating constellations for the use of sailors, could be of no service for the practice of justice.* Here one would expect that he should have shewn us how delineating constellations *not* for the use of sailors, could be of any service to the practice of justice; or what relation there is between *justice* and the *constellations*. But unluckily he has left this short. Now every body knows the use of the constellations in sailing: they had no other way at that time to guide their ships thro' the ocean, but observations of the stars. This use then is obvious; but the other use, which the Professor dreams on, is to seek. As to *Hippo* the daughter, who is allowed to have studied nature in the same manner as the father; the Dr. says, she shewed the will of the Gods, by her oracular answers, or *from the rising of the stars*. The use she made of her knowledge, was to teach the will of the Gods *from the motion of the stars*. This is enough surely to shew that they both studied *Astronomy*. But then this knowledge was applied, it seems, to the purposes of *Augury* or telling fortunes. But can any body (but this Rev. Doctor) be so dull, as to think that two of the greatest Astronomers of the age should apply ALL this knowledge, to such a trifling insignificant purpose; and not rather apply *some* of it least, to the nobler purpose of *Navigation*, for which it is naturally adapted, and very much wanted for that purpose. For without the knowledge of the stars, they could make no long voyages;

ages ; nor had they any other method at that time to guide a ship, but the help of the stars. Consequently it was natural for the navigators to apply to the astronomers for that purpose. But the Professor seems still not to be content in allowing these figures to be the constellations ; for, says he, *perhaps this may not mean to form constellations, but to divide the heaven into regions.* As if dividing the heaven into regions or segments, could teach the *rising and motion of the stars.* *Museus,* he says, *was a poet, and writ a poem concerning the sphere, but there was no sort of reason to conclude that he was a practical astronomer.* I dare say, no body would ever draw such a conclusion, but this sagacious Doctor. For can any body in their senses suppose, that any man will take pains to write a poem concerning a subject, that he knows nothing at all about. How must he describe the circles of the sphere, or shew the nature and use thereof, if he did not understand the sphere ? But all thinking men must rather believe that he knew what he was doing, that he understood the sphere, and of consequence must be an astronomer. I say, the consequence is plainly this, that there was a sphere in the time of *Museus*, or he could not write about it, and that he understood the sphere he was describing ; and if he also made a sphere, as history informs us he did, and was the first that made one ; then this point is fully determined. But if it was only a poem, yet certainly if a man writes upon a subject (especially a mathematical one) he must be allowed to understand that subject, whether he writes in verse or prose. Yet the Rev. Doctor will not allow *Museus* to be an astronomer, because he was a poet ; which is just as good a reason as this, Dr. Rutherford is a *Parson*, and therefore is no *Astronomer.*

He says further, *the accounts we meet with in the poets*

A D E F E N C E O F

Poets (concerning the Argonautic Expedition) is only its fabulous history. And the names of the constellations Sir Isaac has taken from this fabulous history. The poets then cannot be trusted, even to give the names of things truly. But this writer is certainly at liberty to call any thing fabulous he pleases. He may reckon the whole fabulous, if he likes; and that the expedition itself is nothing but a fable. The poets give us a mixture of truth and falsehood, or fables intermixed with truth. This Sir Isaac knew, and by his *philosophical sieve*, could tell how to sift out the true, and leave the fabulous part behind. Whilst such writers as this Professor are forced to take altogether by the lump; and not knowing how to separate them, call the whole fabulous. However, what Sir I. Newton has taken, is mostly from the Historians, who must be supposed to relate matters of fact, and not fables; and these he corroborates from the testimony of the poets. And as far as I can find, this Doctor is of opinion, that the Constellations were formed without having any names given to them; or else what should hinder the names from being transmitted down to posterity, as well as the Constellations themselves. Nay, they could not be described without their names.

The next objection this Rev. Doctor makes is, placing the constellation *Argos* in such a position, that the principal star *Canopus* could not be seen; and he thinks, it should rather have been placed in the zodiac. But any body may consider, that the constellations in the zodiac are of far greater use than any of the rest, as they serve to shew the motions of the sun, moon, and all the planets; and therefore these must be first of all placed; and this being so, he could not place *Argos* in the zodiac without displacing some of the rest, that had been fixt before; consequently it must be placed where he

he could find a vacancy. In the position she was placed, her masts, sails, and rigging might be seen. Besides, it is evidently more natural to place the ship in the horizon, with part of her invisible, as if under water, than have her suspended in the air; as she would have been, had she been set in the zodiac, where sun, moon, and planets move. But let us come to his second principle.

2. *That Cbiron drew the colure of the equinoxes thro' the 15th degree of Aries and Libra, &c.* Now this is a point of history, which is all the *data* allowed, and therefore must be granted. If this Professor disputes it, then he is disputing with *Eudoxus*, and not with Sir I. Newton; for *Eudoxus* was describing the sphere of the antients. But this Doctor tells us, *that he was describing the sphere in his own time*; the consequence of which is, that *Eudoxus* (and even *Thales*) must have lived 940 years before Christ, which is a manifest Falshood. He tells us, that in describing the colure of the solstices, Sir Isaac leaves off, and omits a very material part, which *Hipparchus* adds. This sagacious Doctor, it seems, did not see the reason of this; Sir Isaac had *data* enough without going any further, to determine what he wanted. And if the remaining description was as it stood in *Eudoxus's* time (as this writer says); then what had Sir Isaac to do with that? for the position of the colures must be quite altered in near 600 years. But the Doctor seems not to understand, that many of the constellations had been misplaced, and drawn otherwise in succeeding ages, and could not well be rectified; and such as were rightly placed, or could be rectified, Sir Isaac made use of. Besides, if this Objector knew any thing in Astronomy, he would easily understand, that such stars as are placed near the pole, or far from the ecliptic, are subject to great errors; for a small one there, becomes a very

very great one at the ecliptic, and therefore were judiciously omitted by Sir Isaac ; and for this reason, he need make no wonder of these being omitted. Besides, where the place of the colure is too generally defined, and a star cannot be found to direct it with sufficient exactness ; such *data* as these ought to be left out. For any body that knows what he is about, will choose clear *data* before ambiguous ones ; while such a judge as this Doctor must take all without distinction. In order to determine this point, Sir I. Newton, with uncommon sagacity and judgment, has in either case, chosen five of the best defined, and most unexceptionable places, to found his calculations upon ; which he has compleated with as much skill and nicety, as the solution of such a problem deserved. So that the method he has followed will be admired and approved by all proper judges ; tho' it be condemned by such ignorant, perverse critics as this, that are too dull to see the reason and beauty of such a proceſs.

By this judicious way of proceeding, he has found the mean places of these colures (from these five), to be *Taurus* $6^{\circ} 29' 15''$, and *Leo* $6^{\circ} 28' 46''$, which are so surprisingly near, that the most scrupulous cannot expect any thing nearer ; which is a full testimony of its truth. So that at a mean, the four cardinal points had gone back $1^{\circ} 6' 29''$, which amounts to 2627 years. Nay further than that, those very places which this Professor says he left out, he shews that they all fall in the solstitial colure, as *Eudoxus* had described them ; and consequently that this had all the characters of that colure, according to that description. But this purblind critic either could not or would not see all this. This being all made plain, will he say, that *Eudoxus* lived so many years since ? Yet he says as much, when he says *Eudoxus* described the sphere

sphere as it was in his own time. But as far as I can see, he will neither allow the colures to pass thro' the middle of the signs in *Eudoxus's* time, nor *Chiron's* time, nor any time else; so strangely full of contradictions is he.

What he says about the little Bear is nothing to the purpose. If it was not formed into a constellation in *Chiron's* time, what had Sir I. Newton to do with it. And even if it was, it was too near the pole to make any calculations upon; but any way, *Eudoxus* could not describe the path of the colure in other words. But if the Professor would have done any thing to the purpose, he should have calculated how far this star was from the colure; and then a better judgment could be made. He likewise says, *it is impossible to conclude from this description of the cardinal points, that Chiron placed them in the middle of the constellations.* Now had the Doctor made any calculations to shew this impossibility, or to shew how far any of these points deviate from the colure, he had done something. But he seems to be better qualified for forging arguments, than making calculations. But Sir Isaac has actually made calculations, and has found and demonstrated all these descriptions to be right, by the surprizing agreement among all these observations; which is sufficient to overthrow all the Doctor's wild guesses, and silence all his cavils at once. And indeed it is no wonder to find him mistaken in a matter, which is quite out of his sphere; when he mistook in a matter within his own sphere, about the nature of *Virtue*; till a Lady, Mrs. Coburn, set him right. *O Doctor!*

3. We come now to his third grand article, *whether they knew how to draw the colures exactly.* This is such a childish objection, that I don't know what answer is fit to be given to it. If he means perfectly exact, there is no man at this day can do it.

A D E F E N C E O F

For at this rate the colures are to be drawn precisely thro' such and such stars, which can be looked on no otherwise, than as so many mathematical points, which cannot be. All that can be concluded from hence is, that each colure, when rightly drawn, will pass so near any star, that the difference cannot well be discovered, by such instruments as were then in use; or that such and such remarkable stars, come the nearest thereto of any. But let us see how he argues upon this point. He tells us, *Eudoxus was describing the sphere as it stood in his time*; and in the very next words he confutes himself by telling us, *we are sure that he placed them wrong, for we can prove that they were not in the middle of the constellations in his time.* Was ever poor Doctor so embarrassed with an argument; he first tells us, that *Eudoxus was describing the sphere in his own time*; and immediately after, that such a description did not at all agree with the sphere at that time. Would not any body but this Objector then conclude, that it was not described for that time, but for some other. Every circumstance leads him, as it were, by the nose, into a direct path, and yet he most strangely misses his way. Instead of doing that, he has nothing to recur to, but that ridiculous shift, that *Eudoxus did not know how to draw the colures.* Here then it seems we have a famous Astronomer, and the greatest of the age, that did not know how to draw one great circle perpendicular to another on the sphere, he was so very ignorant. I know not what this Doctor may think, but I believe all the world will laugh at so idle a chimera. But the Doctor keeps fluttering about among these absurdities, not knowing where to fix; but will readily agree to any impossible scheme, rather than to Sir *J. Newton.* Here one would think, if *Eudoxus* had missed his aim in drawing thro' one star, he would scarce have missed

missed two ; and if he missed two, he would hardly miss three ; but by strange ill luck, he has *missed* them all, according to him. And yet Sir Isaac has shewn that he has *bit* them all. So that this great Professor has made a fool of *Eudoxus*, or else of himself. But Professor *Simson*, of *Glasgow*, gives a quite different account of *Eudoxus*; he tells us, that it is the opinion of some learned men, *that Eudoxus was the author of the chief propositions in the 12th book of Euclid.* See his notes on B. 12. Euclid.

But to shew you how great an Astronomer this Professor is, he quotes *Hipparchus* for saying, that no secondary of the equator, could pass thro' all the places, before mentioned to be, in the colures. Now if this Gentleman knew any thing of the sphere, he would not at all wonder at this ; because it is impossible, that the two great circles (the two colures) which were perpendicular to the equator in the time of *Chiron* ; I say, it is impossible that the same two great circles should be perpendicular (or secondaries to) the equator, in the time of *Eudoxus* : For *Hipparchus* knew better. And this is another proof, that the sphere described was not for the time of *Eudoxus*, but for some other preceding time. And hence this Rev. Doctor need take no further pains to make *Eudoxus* an ignoramus, for it will all fall upon himself ; all his arguments being founded upon his own ignorance ; and all the consequences, he draws from his absurd reasoning, must all fall at once. But where is his modesty, thus to fall foul of a person of such transcendent abilities as Sir Isaac ; and who has obliged the world with so many eminent discoveries, and which were thought before to be beyond human art ; and yet with no better success, as he himself appears to be no judge of it ; and with no better design than to degrade that great man, who shines brighter for being cavilled at. For the same spirit and

and genius that runs thro' his other noble works, are equally visible in this of his *Cbronology*.

I think it needless to pursue this learned Doctor any further ; it is plain he did not understand what he was writing about : every incident confutes him. But Sir Isaac stands firm, his plan is confirmed not only by the agreement among the observations, but also by a great many other proofs of a different nature, all conspiring to prove the same thing. And in such cases as this, which are not capable of mathematical demonstration ; the concurrence of so many circumstances must make the thing so highly probable, as to take away all doubt and scruple, and give entire satisfaction.

After it was known that Sir *J. Newton* had writ a book of *Cbronology*, a foreigner (one *Abbe Conii*) under the pretence of friendship to Sir Isaac, obtained a copy of the Chronicle or Index, with a solemn promise to keep it secret. But observe how he kept his promise ; he communicated it to one *Souciet*, a jesuit, who got a bookseller at *Paris* to print it, without Sir Isaac's knowledge or consent ; and with it a pretended confutation of the Chronicle, by the said *Souciet*. This was sent to Sir Isaac, who confuted every article of this pretended confutation. So that this may be said to be stolen from him, much after the manner that *Leibnitz* stole his Fluxions ; but with a worse design, to involve Sir Isaac in disputes to oblige *Leibnitz*.

And the same *Souciet*, after Sir Isaac's death, writ another Tract against him. But Dr. *Halley* shewed him, that thro' his ignorance in Astronomy, he had made false calculations ; and had assumed a wrong star, for the first star of Aries ; and shewed him when all these things were rectified, the matter would be just as Sir *J. Newton* had described it. Thus you see how this great man has been used.

Of Dr. BEDFORD's CHRONOLOGY.

THIS Author is one of the same stamp as the foregoing, or rather more enthusiastical ; for if you will believe him, he can tell the very day when the most remarkable events in the beginning of the world happened. He promises a large book with maps, wherein, no doubt, he will inform you of the very day when the world began, where the garden of *Eden* was placed, where the Ark rested, and how every Spot of Ground was peopled after the Flood, &c. And all such difficulties as are without the power of Sir *I. Newton*, or any body else besides. For in the Advertisement to the present book, he tells us, that the sun was created in the Equinox on a *Thursday*, and the moon at full ; that God spoke to *Noah* on a *Sunday* ; that *Noah* ceased from bringing the living creatures into the Ark, on a *Sunday* ; that God spoke to him to come out of the Ark on a *Sunday* ; that God spake to *Abraham* on a *Sunday* ; that *Solomon* dedicated the temple on a *Sunday* ; that the angels sung Glory to God, for the birth of Christ, on a *Sunday*, &c. &c. So that our Author, to be able to solve such problems as these, without any *Data*, cannot be less than inspired ; and so I believe he is, with presumption and vanity. But I think few people, who are thus glutted with such impossibilities, will think it worth their while to penetrate any further into this profound Treatise ; in which he is as confident, as if he had been present at every event. I shall therefore only give a few specimens of his fine reasoning.

He begins his book with a most fulsome *Encomium* upon Sir *I. Newton*, as a great Mathematician ;

cian ; tho' he was no manner of judge about it, as will be seen after. He tells you, that Sir Isaac's Chronology was writ in his dotage ; tho' Sir Isaac expressly says, he did it when at Cambridge, and therefore when he was a young man ; and he only revised it when he was old. And in reality it had been the work of his whole life-time, at his vacant hours. This Author tells us, the reason that Sir Isaac would not publish it in his life-time was, that he was conscious of his mistakes and errors. And he may as well say, that his refusing to print his *Optical lectures*, or his book of *Optics*, was because he was conscious of their being erroneous. So ready are such authors as this to give false reasons for things, and miss the true ; for the true reason was, to prevent wasting his time in useless disputes, with weak and obstinate persons, not worth a dispute, like this Author.

This Doctor seems to be under great concern, that Sir Isaac's Chronology should differ from the Scripture Chronology ; but I believe it is rather for differing from three or four bishops he mentions. For Sir Isaac takes care all along to connect his account with that of the Scripture ; tho' perhaps not according to the sense of this Author. And because Sir Isaac differs from all other Chronologers, therefore he thinks it must needs be false, and so he condemns it at once, because not agreeable with him and his four Bishops.

It is no news to tell us, that Sir *I. Newton's* Chronology differs from all the world ; it was Sir Isaac's professed design. He attempted to correct the antient Chronology, which no body else could do, or at least has done. But this Rev. Doctor gives us nothing but a *botch potch* of the common Chronology, with all its inconsistencies and errors ; which he knew not how to get rid on, or to mend. This he intermixes with his own wild conjectures ; and

and a romantic account of his own, taken at random from all sorts of authors. But let us see how he goes on.

Arguing against Sir *J. Newton* about ship building. He tells us, that *the first pattern for ships was the ark, which was made square, that it was not designed for sailing, but to lie on the water.* How comes it then to be a fit pattern for ships, that are to sail from one place to another? And he tells us, *that tall and long ships were probably invented by Ham, who having been in the ark, could not be ignorant of navigation.* As if a man that was close shut up in a trunk, which was to lye upon the sea, could from that situation, conceive any notion of navigation: so little knows he what *Navigation* means.

Sir *Isaac* says, the Chronology of things done in Europe above 80 or 100 years before *Cadmus* (who invented letters) cannot be admitted. This Doctor replies, *and the history of Moses, of the things done above 100 years before his time, will be destroyed by the same argument.* Here he puts *Moses* upon the same footing as other common historians, he owns no difference of circumstances.

Sir *Isaac* tells us, that *Sesac* left geographical tables of all his conquests at *Cholchos*: and there *Geography* had its rise. This Objector has nothing to answer but, *it is very seldom that such arts are found out by martial men, who have something else to do.* Here is another instance of his ignorance; for can any man invade a country, but he must know or find out the situation of places, how they are bounded by one another, or by the sea, &c. And the knowledge of this is *Geography*. And his ignorance is still further confirmed in this, where he makes *Sibor* not to be the *Nile*, but a river in the country of the *Amalekites*, where there is no such river. When he writ this, he certainly was *not capable of thinking*.

A D E F E N C E O F

Sir Isaac says, the temple built by Menes could not be above two or three hundred years older than Pjanimiticus that finished it. This Doctor answers, *Cbronology may be reduced still further; for Herod the great, who was alive at the birth of Christ, finished Solomon's temple; and therefore it could not be above two or three hundred years older than the reign of Herod.* So that this blundering writer cannot tell the difference between *finishing* and *repairing*. There are endless examples of such wretched stuff as this given for reasons and arguments, and hardly any thing better.

His representing Sir I. Newton as not capable of knowing what was demonstration and what was not; is very conformable to the sentiments and disposition of such a brute as this, that has not the least idea of the nature of a demonstration. Now for some strokes of his astronomical genius.

The first thing of this sort that he presents us with, is the calculation of the sun's place 2066 years before Christ, which place he falsely makes in *Capricorn* $14^{\circ} 8' 19''$; which should be *Sagitary* $13^{\circ} 23' 20''$. The Egyptian year did not take place 1319 years before *Nabonassar*; and if it did, this absurd calculation proves nothing at all. Yet from this, he makes the distance of the sun from the tropic only $45'$, when he is 21 days from it, by his own calculation. Nor does it appear by what sort of legerdemain, he makes $45''$ answer to 21 days. Also it may be observed, from his process, that he makes the Julian year take place from the Era of *Nabonassar*; which is another blunder.

When the Julian year begun, he tells us, the beginning of the year was, when the sun was in *Capricorn* $8^{\circ} 50' 57''$, and its distance from the tropic $18'$; so by this, the tropic was in about $8^{\circ} 33'$ of *Capricorn*, which is an absurdity.

He makes a remark from *Scaliger*, that the co-lure

lure could not pass thro' the tail of the serpent, altho' there is no calculation made about the serpent; so that this is nothing to the purpose. Yet he concludes that the colure had been placed wrong, just the same as Dr. *Rutherford* says, an astronomer as wise as himself.

This author tells us, *the mariners at sea find out the latitude of the place, not from the exact height of the sun at noon*; and from such stuff as this, he tells you that *Chiron (or some other) would place the summer solstice and its colure, not at the entrance of Cancer itself, but before it.*

In quoting *Hipparchus*, he tells us *Hipparchus well knew there was 1180 years between the time of the argonautic expedition, and the discovery of the motion of the equinox.* But certainly the author does not well know what he writes, for if *Hipparchus* had known this so well, he would have told us so. And all the consequence he draws from this supposition is nothing but a heap of absurd nonsense.

The next astronomical flourish he makes, is to calculate the time of *Arcturus*' rising; but before he begins, he has his way to pave with so many *if's* and *and's*, and *suppositions*, that the reader must be surfeited before he begins his calculation. Then he assumes the lat. of *Cuma* (but it was *Ascra* where *Hesiod* lived), to be $38^{\circ} 54'$, which should be $39^{\circ} 20'$ at least, but no body can tell exactly; but a small error will do him a deal of service, and he can apply it his own way. In calculating from his data, he finds the declination of *Arcturus* an. 870 before Christ, to be $34^{\circ} 25'$, which should be $34^{\circ} 19' 32''$; and his right ascension $179^{\circ} 32'$, which should be $178^{\circ} 21'$; and so finds *Arcturus* at sun-set, $2^{\circ} 56'$ under the horizon, which should be $1^{\circ} 46'$, by his own data, if he had wrought right. Now this eminent astronomer, or his computer,

puter, has made no manner of allowance for refraction ; which being done, it will bring *Arturus* very near the horizon, as it ought. Nor does he take notice of refraction in the rest of his sublime calculations that follow. But to make all clear, he tells us, that the number 60 may stand for 55, or 65, &c. which is a *poetica licentia*, that I did not understand before.

This doctor tells us Sir Isaac's whole book is built upon this, that *Chiron* made a sphere, &c. But this is a most impudent falsehood, for the whole book is not built upon this ; it stands upon many more pillars, which this writer, with all his sophistry, is not able to demolish. Whether *Chiron*, or who-ever made the sphere, it will be the same thing ; if it was made at that expedition, or a little after, as there is no room to doubt.

This Rev. Doctor contends that the ages of men were longer in ancient times than at present ; and therefore he thinks Sir Isaac has made them too short. But he cannot make it out, that they were any longer within the bounds of this chronicle. And *David* will be against him ; for he tells us, not of one particular man's age, but in general, that the age of man is 70 or 80 years, as at present,

And the like objections he makes against the reigns of kings ; tho' Sir Isaac has confirmed the thing by so many catalogues of vast length. And this Author's picking out short catalogues of greater lengths, whether of ages or generations, is nothing to the purpose. He may as well take one age or one generation, as a standard of all the rest. But this is not doing justice to the cause.

This worthy Doctor is so enraged, that Sir I. Newton's Chronology is opposite to all the world besides ; that he damns it at once by wholesale ; and falls into such a strain of railery as is hardly to be

be matched in all Billingsgate. He can talk of nought for several pages together but *contradicting the Scriptures*. This system of Chronology giving men a *dismal tincture*, rooting-out of religion, bringing in *infidelity*, that such *poison* ought not to go abroad; the Devil's spite against the *sacred oracles*, and the *mustering* of his *forces*, *blasphemies*, *heresies*, liberty for *new religions*, *Atheism*, *Deism*, *Hell torments*, man's *immoralities* and *debaucheries*, attacks on *Christianity* and *Scripture Chronology*; *smoaks* arising out of the *bottomless pit*, that *darken the sun and the air*, &c. &c. and, I believe, by the time that the reader has got thro' it all, he will be nearly choaked with all this *smoke*. And all this Rant is, because men will not swallow every stupid thing, that such men would cram down their throats. After so pathetic an harangue, no body need have any doubt about his pacific disposition; this pious Doctor, I believe, will hardly send any man to heaven, that should presume to give any credit to such a vile book as Sir *I. Newton's Chronology*. But I understand not why all this stuff is introduced here into a book of Chronology; except this pious Doctor meant to call down *fire from heaven* upon all such wicked persons, as should take the liberty to differ from him in *Cronological matters*, as well as in *Religion*.

He frequently mentions Sir *I. Newton* as a great and eminent Mathematician, nay, the greatest in the world. Yet he dare not trust him with the solution of certain astronomical problems; but attempts to give more exact solutions to them himself. At this rate he must be a greater mathematician, and a *greater* than the *greatest* in the world. Or perhaps Astronomy must rather be reckoned a part of Divinity, than of Mathematics.

However compleat he may be in the mathematical arts, he is certainly compleat in the art of

Squab-

Squabbling. For example, Sir Isaac tells us, that the observations of the ancients were but *coarse*. This word is of infinite service to him, and he *ebows* it over and over. By the *magic* of this word he can make any error, great or small, serve his turn. For if he misses his mark never so far, he can always account for it from the *coarse* observations of the antients. This with him is a word of great latitude, and has no bounds.

I suspect what irritates his temper most of all is, that this regular system of Sir Isaac's is likely to overturn his great *Babel*; which makes him labour with might and main to secure it; but with more *pains* than *prudence*. For there is as much difference between them, as between a regular and beautiful garden, and a confused wilderness.

But to take my leave of *this Author*, this renowned Champion for vulgar errors; Sir Isaac having proved *Osiris*, *Bacchus*, and *Se/ostris* and *Sesac*, to be all one and the same person. As each of them was king of all *Egypt* at the very same time, and by a great many other circumstances common to them all. This acute Reasoner argues thus; *there were three famous men in the world, Hannibal, Bellisarius, and Churcbl; they were all very eminent generals, remarkable for their courage and conduct, led their armies into foreign countries, fought many battles, took many towns, &c. therefore they are the same with the Duke of Marlborougk.*

But pray stop a little Rev. Doctor, did all these men go into the *same* foreign countries, fight the *same* battles, take the *same* towns, &c? if not, where is the similitude? Now, Sir, as soon as you can make out, that these four men were each a general of the *whole* army, at one and the *same* time, in one and the *same* place (as the parallel requires); then in spite of your *Logic*, they will be one and the *same* man, or rather four different names

THE CHRONOLOGY.

157

names for the *same* man. This Doctor is certainly very unhappy in drawing parallel cases, to chuse such where all the circumstances are different, instead of being the same or like. But for all that, the conclusion will be the very same with him, whether the circumstances be the same or different. Does not his scarlet blush at this! Could any body of common sense offer such childish things to the public. He certainly has a mean opinion of his readers, to think they can be thus imposed on. Yet all his parallel cases are of the same kind.

F I N I S.

E R R A T A.

Mensuration, p. 104, l. 2b. read $\frac{7854}{3} \times CD$, is the

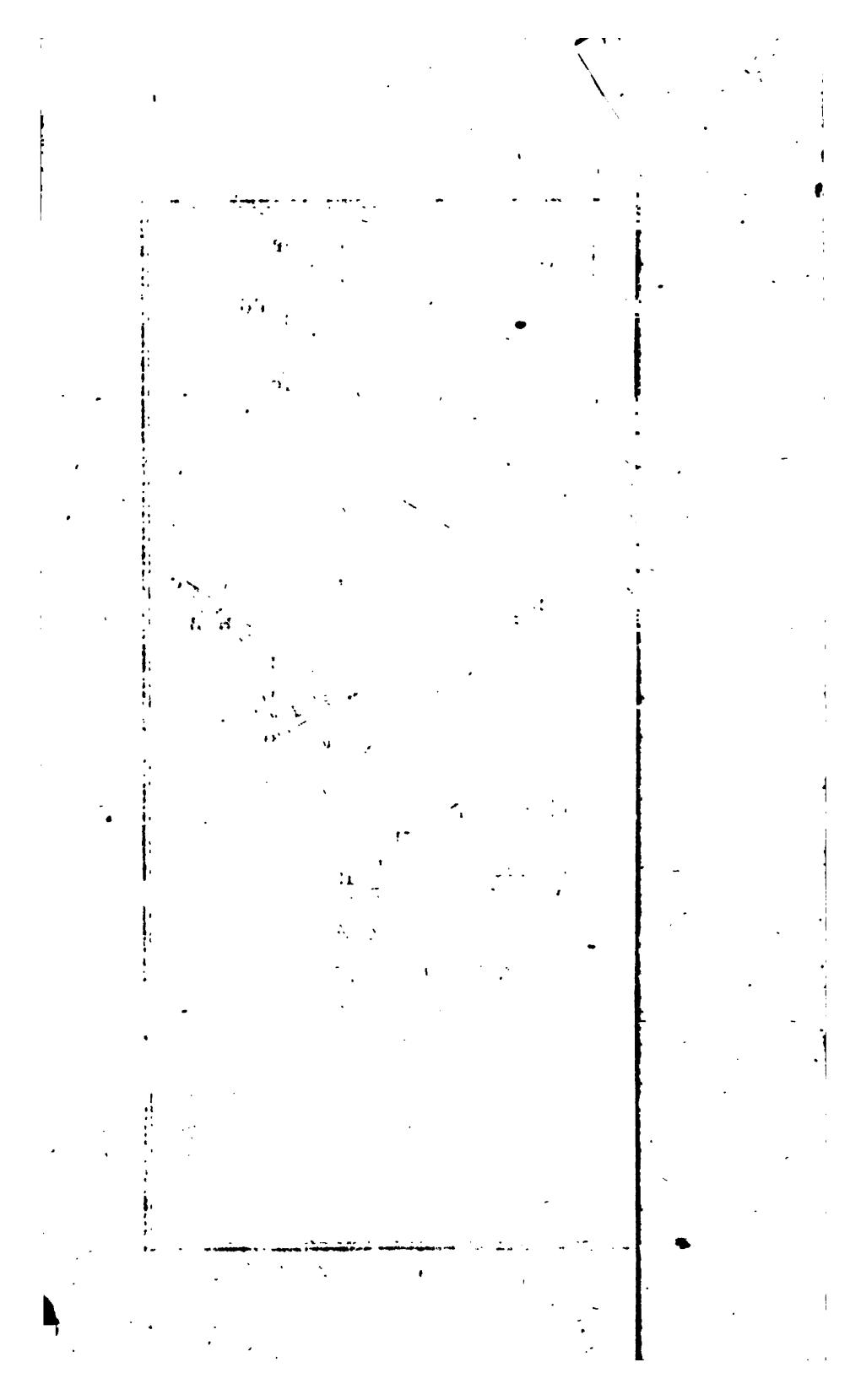
Surveying, Preface, p. ii, l. 3b. 4b. read $\mu\gamma\mu$

E R R A T A.

b signifies, from the bottom.

page	line	read	page	line	read
5	2b	SA or TA;	16	1b	IH : HF :: (iX : XY
6	o pa. 6.		17	1	dele fig. 20.
9	7b	$Sp^3 \times Pv^3 =$	19	3b	: s : PM =
	6b	And $Sp^3 \times PV^3 =$	29	13b	d in the same
10	1	round R, in P ::		2b	de, fg touch
	2b	rectangle uPv.]		1b	from C on
	1b	$Qv^2 + Pv =$	30	3	of B in d ::
11	2	$Pv \times uV + uPV = VPv$		14	thro' $\beta\gamma$
	4b	Conics Prop. 24. Ellipsis)		16	A as C β
		perpendicular SY		24	: Cy — $\gamma\delta$
	3b	(fee alfo Cor. 7.)	87	1	AZ, Cz
12	1	dele fig. 5.	15	1	$\mu : : \rho\sigma :$
	12	[Pr. 21.] three—lines. If	105	15	G — C
	21	Lem. 16.	109	16	$\overline{G — Q} =$
	24	in the same			$\overline{G — \gamma}$
13	6	fig. 7.	Pl. I		fig. 6. for u write n.
	18	PI : : fine of			fig. 4, 11, 12, for e write c.
	22	tangent at A,			fig. 13. draw the prick
14	6	dele fig. 11.			lines OB, OeD.
	10	fig. 11.	Pl. III		fig. 29. for n write w.
	11b	And kn : ga ::			fig. 36. for g write q.
	3b	Oe : OD ::			
15		dele fig. 13.			

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